1. Write the augmented matrix of the system of linear equations, bring it to the reduced row-echelon form, and find all solutions of the system

$$x_1 \qquad + 2x_3 + 2x_4 = 7$$
$$2x_1 + x_2 + 5x_3 + 5x_4 = 19$$
$$x_1 \qquad + 2x_3 + 3x_4 = 10.$$

Solution. We have

$$
\begin{bmatrix}
\overset{x_1}{1} & \overset{x_2}{0} & \overset{x_3}{2} & \overset{x_4}{2} & | & 7 \\
2 & 1 & 5 & 5 & | & 19 \\
1 & 0 & 2 & 3 & | & 10
\end{bmatrix}
\longrightarrow
\begin{bmatrix}
1 & 0 & 2 & 2 & | & 7 \\
0 & 1 & 1 & 1 & | & 5 \\
0 & 0 & 0 & 1 & | & 3
\end{bmatrix}
$$

$$
\longrightarrow
\begin{bmatrix}
1 & 0 & 2 & 0 & | & 1 \\
0 & 1 & 1 & 0 & | & 2 \\
0 & 0 & 0 & 1 & | & 3
\end{bmatrix},
$$

which is the reduced row-echelon form of the augmented matrix of the system. There is one free variable x_3, so the solutions are $x_3 = t$, $x_4 = 1$, $x_2 = 2 - t$, $x_1 = 1 - 2t$, where t can be any number.

Answer. The solutions are $x_1 = 1 - 2t$, $x_2 = 2 - t$, $x_3 = t$, $x_4 = 3$, where t can be any number. In vector form:

$$
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4
\end{bmatrix}
= t
\begin{bmatrix}
-2 \\
-1 \\
1 \\
0
\end{bmatrix}
+
\begin{bmatrix}
1 \\
2 \\
0 \\
3
\end{bmatrix},
$$

where t can be any number.