

2. Find all values of the parameters  $a$  and  $b$  for which the system

$$\begin{aligned}x + ay &= 1 \\ 2x + 3y &= b\end{aligned}$$

- a) has no solutions
- b) has a unique solution
- c) has infinitely many solutions.

**Solution.** Writing the system in the matrix form and reducing it, we get

$$\left[ \begin{array}{cc|c} 1 & a & 1 \\ 2 & 3 & b \end{array} \right] \longrightarrow \left[ \begin{array}{cc|c} 1 & a & 1 \\ 0 & 3 - 2a & b - 2 \end{array} \right].$$

Now, if  $a \neq 3/2$  then  $3 - 2a \neq 0$  and we can find  $y$  from the second equation as  $y = (b - 2)/(3 - 2a)$  and then find  $x$  from the first equation as  $x = 1 - ay$ . Hence in this case the system has a unique solution.

If  $a = 3/2$  and  $b \neq 2$  then the system has no solutions since the last equation reads  $0 \cdot x + 0 \cdot y = b - 2 \neq 0$ .

If  $a = 3/2$  and  $b = 2$  then the last equation reads  $0 = 0$  and the system has infinitely many solutions since we can let  $y$  be any number and then find  $x = 1 - ay$ .

**Answer.**

If  $a = 3/2$  and  $b \neq 2$ , the system has no solutions.

If  $a \neq 3/2$ , the system has a unique solution.

If  $a = 3/2$  and  $b = 2$ , the system has infinitely many solutions.