2. Suppose that $AXBC = D$, where A, B, C, and D are $n \times n$ invertible matrices. Express X in terms of the matrices A^{-1}, B^{-1}, C^{-1}, and D.

Solution. We have

$$\begin{aligned}
AXBC = D &\Longrightarrow A^{-1}(AXBC)C^{-1} = A^{-1}DC^{-1} \\
&\Longrightarrow \left(A^{-1}A\right)XB\left(CC^{-1}\right) = A^{-1}DC^{-1} \\
&\Longrightarrow IXBI = A^{-1}DC^{-1} \\
&\Longrightarrow XB = A^{-1}DC^{-1} \\
&\Longrightarrow (XB)B^{-1} = \left(A^{-1}DC^{-1}\right)B^{-1} \\
&\Longrightarrow X\left(BB^{-1}\right) = A^{-1}DC^{-1}B^{-1} \\
&\Longrightarrow XI = A^{-1}DC^{-1}B^{-1} \\
&\Longrightarrow X = A^{-1}DC^{-1}B^{-1}.
\end{aligned}$$

Answer. $X = A^{-1}DC^{-1}B^{-1}$.