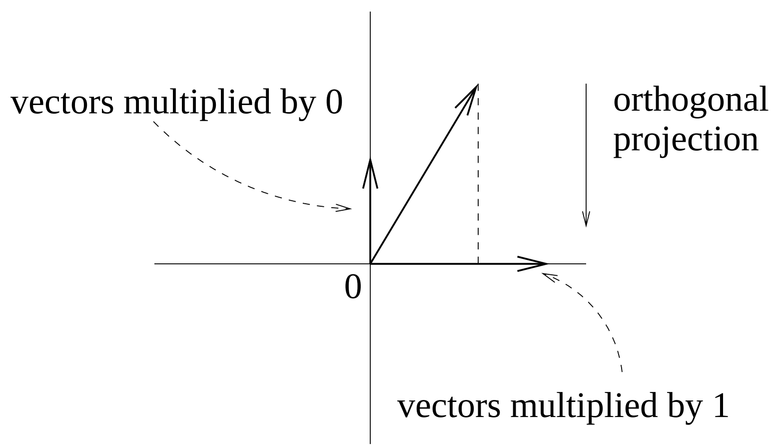
**2.** Consider a linear transformation $\mathbf{R}^4 \longrightarrow \mathbf{R}^4$ that is the orthogonal projection onto a plane in $\mathbf{R}^4$. Let $A$ be the matrix of the transformation. Find the eigenvalues of $A$ and describe the eigenvectors of $A$.

**Solution.** We are looking for vectors $\vec{v} \neq \vec{0}$ such that $A\vec{v} = \lambda\vec{v}$ for some number $\lambda$. Geometrically, we are looking for non-zero vectors that get multiplied by a number (stretched or shrunk if the number is non-negative, or flipped about the origin and then stretched or shrunk if the number is negative) by the orthogonal projection.




By inspection, there are two families of such vectors. The first family consists of the non-zero vectors in the plane we are projecting onto. All those vectors are multiplied by 1. The second family consists of the non-zero vectors in the orthogonal complement of the plane (which is another plane). All those vectors get multiplied by 0.

**Answer.** The eigenvalues are 1 and 0. The eigenvectors with the eigenvalue 1 are the non-zero vectors in the plane onto which we project. The eigenvectors with the eigenvalue 0 are the non-zero vectors in the orthogonal complement of the plane.