Math 115 Group Homework 1
Fall, 1999

1. Suppose that an astronaut makes the following lunar journey. After liftoff from Cape Canaveral, her spacecraft orbits the earth three times before her rocket engines are restarted briefly to take her out of orbit and send her on her way to the moon. Earth's gravity keeps slowing her down until she is most of the way to the moon, at which point the moon's gravity becomes dominant and she starts speeding up as she falls toward the moon. When she is near the moon, she restarts her engines for a brief time to put her into lunar orbit, and then orbits the moon four times before burning her engines for the last time to send her back toward earth. When she reaches earth, she does not orbit but instead heads straight for her landing in the Pacific Ocean. Plot a graph of her distance from the surface of the earth as a function of time, from the moment of takeoff until landing.

   NOTE: Though it is not stated explicitly as part of this exercise, you should clearly label the axes and mark points of interest on the distance axis. You should also include a clearly written explanation, in complete sentences, of the behavior of the graph along each segment. Be sure to define every variable you use; for example, if you use the letter d to represent distance, say so. (Incidentally, you should also use meaningful letters for variable names you invent. The letters x and y are badly overused. You almost never run into them as variable names in the real world.)

   Your group homework this semester will be graded not just for correctness of results, but also for clarity of exposition. Thus, including just the bare minimum amount of work that gets the correct answer onto the page will usually not result in a good grade. You should also include clear explanations, written in complete sentences. Be sure to incorporate graphs or tables wherever they help clarify your results, even if the instructions for the exercise do not explicitly call for them. The point to these exercises is not just to show that you can get the correct answers, but also to demonstrate that you can explain your results clearly and completely.

2. The following table gives the radius of the orbit of each of the Sun's planets, along with the period of revolution of the planet around the Sun, in years.

Planet	Orbital Radius (AU)	Orbital Period (Years)
Mercury	0.3871	0.2408
Venus	0.7219	0.6152
Earth	1.000	1.000
Mars	1.520	1.874
Jupiter	5.201	11.86
Saturn	9.539	29.46
Uranus	19.19	84.01
Neptune	30.06	164.8
Pluto	39.60	247.7

An AU, or astronomical unit, is a distance equal to the orbital radius of the earth, about 150,000,000 kilometers.

An important principle of orbital mechanics is that the square of the orbital period of a planet about the Sun is proportional to the cube of the orbital radius. Show how the data in the table illustrates this principle. What is the constant of proportionality in this case? Why are the units used in the table so convenient?

3. It is often important to compute the value of your assets for tax purposes, where the value of an asset such as a car depreciates with time. One common method of computing the value of such an asset is *straight-line depreciation*, in which the value of the asset is assumed to be a linear function of time. Suppose that you buy a car for $14,000 and that you use straight-line depreciation to compute its value over time, with the value assumed to fall to 0 after seven years.

   (a) Give an algebraic interpretation of this relationship by finding a formula for the value of the car as a function of the time since its purchase. (In the spirit of the comments about completeness and clarity at the end of Exercise 1, you will want to make certain that you define each variable you use and state the units in which it is measured.)

   (b) Give a graphical interpretation of this relationship.

   (c) Give a numerical interpretation of this relationship by constructing a table.

   (d) Given the real-world nature of this problem, find the domain and range of the function whose formula you found in (a).

   (e) Interpret in real-world terms the meaning of the slope and each of the intercepts of the graph you found in (b).

4. In 1980, a personal computer of average capability (at least, average by the standards of the time) could be purchased for $4000. In 1999, a personal computer of average capability costs about $1200.

   (a) Suppose that the relationship between the cost of a personal computer of average capability and the number of years since 1980 is linear. After selecting appropriate variable names, find a formula for the relationship.

   (b) Now suppose instead that the relationship is exponential. Find a formula for the relationship.

   (c) Which model do you think is more realistic?