**Math 115 Group Homework 10**
**Fall, 1999**

This group homework set consists of a project designed to show you an application of integrals to a common type of problem in computing masses. As always, in this exercise you should use only material and techniques you have learned so far in this course.

Suppose that you have a sphere of radius 18 centimeters that is made of material of variable density, so that the density of the material $r$ centimeters from the center of the sphere is $10e^{-r/10}$ grams per cubic centimeter. The purpose of this exercise is to find the mass of the sphere.

(a) Think of the sphere as if it were an onion with $n$ layers, each having the same thickness $\Delta r$, and that the layers are numbered $1, 2, \ldots, n$ as you go from the center of the sphere outward. For $i = 1, 2, \ldots, n$, let $r_i$ be the distance from the center of the sphere to the outer edge of layer $i$. What is the area of the outer surface of layer $i$ in terms of $r_i$? (Though you should of course answer in one or more complete sentences, no explanation is necessary. It might help to look through the formulas on the last few pages of your book.) Be sure to state the units in which your expression measures the area.

(b) The volume of layer $i$ should be approximately equal to the expression you found in (a) multiplied by $\Delta r$. Why? Why is this not *exactly* the volume of the layer? Would you expect this approximation to be better if the layer were thick (that is, if $\Delta r$ were large) or thin (that is, if $\Delta r$ were small)? Why?

(c) Keeping in mind that mass equals density multiplied by volume, find an expression for the approximate mass of layer $i$ in terms of $r_i$ and $\Delta r$. What additional consideration makes this, in a sense, even more of an approximation than the volume approximation found in (b)? Would you expect this approximation for the mass to be better if $n$ were small or $n$ were large? Why?

(d) The expression you just found should have the form $f(r_i)\Delta r$, where $f$ is some function. Using sigma notation, write an expression in terms of $f(r_i)$ and $\Delta r$ for the approximate mass of the entire sphere. Would you expect this approximation to be better if $n$ were small or $n$ were large? Why? What would you expect would happen as $n \to \infty$?

(e) Write an expression in terms of some integral that you would expect to equal the mass of the sphere. Explain.

(f) By considering the derivative of $f$, decide whether $f$ is strictly increasing, strictly decreasing, or neither, for domain values between 0 and 18.

(g) Use your RSUMS program to find the mass of the sphere accurate to the nearest 100 grams. Explain why you can be certain you have that much accuracy.

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