Math 115 Group Homework 3
Fall, 1999

1. One sheet of standard office paper is about one-tenth of a millimeter thick. It is about 385,000 kilometers from the earth to the moon.

   (a) Suppose that you start with a sheet of standard office paper and fold it in half to double its thickness, then fold the resulting doubled sheet of paper in half to double the thickness again, and keep repeating this process so that the total number of times you fold the paper is some number n. Write a formula of the form $T(n) = Ae^{kn}$ for the thickness $T(n)$ of the folded paper, *in meters,* after doing all this folding. Use exact values for the constants A and k, rather than approximations.

   (b) Assuming that you can keep folding the paper as many times as you like, how many times must you fold it before its thickness reaches from the earth to the moon or beyond? Is this in some way surprising?

   (c) Assuming that you must fold the paper at least once and that there is no upper limit on the number of times you can fold it, what is the domain of the function T? (Think about this carefully!)

   (d) What would you consider to be the practical domain of T? Attach your evidence to your homework paper.

2. Suppose that you want to take mathemagic lessons from Max, the world-famous Mathemagician. When you ask Max how you can become his student, he replies as follows.

   "To be my student, you must show me that you have some mathemagical ability already. I am thinking of a function f that is both odd and even and whose domain is all real numbers. I am also thinking of a number x. If you can read my mind and tell me the value of $f(x)$, then I will accept you as my student.

   "If you need some hints, then I will sell you either the formula for f or the value of x for $2000, and as today's special deal, I will sell you both for $3000. I do accept Visa and Mastercard."

   How much should you pay Max?

3. Suppose that in a laboratory devoted to growing a certain type of genetically engineered plant, the light level varies sinusoidally over a twenty-four hour period, being completely dark at midnight. Let t be the number of hours since midnight of the first full day the laboratory was in operation, and let L be the light level in the laboratory in some units.

   (a) The relationship between L and t can be described by either of the formulas $L = D + A\sin(B(t-C))$ and $L = D + A\cos(B(t-C))$. Which is more natural to use in this situation, and why?

(b) Rewrite the formula you decided in (a) is the more natural one to use, substituting numbers for the letters A, B, C, and D wherever you can determine actual values for these constants.

(c) If there is some relationship between the remaining constants for which you cannot determine actual values, then use that relationship to rewrite your formula from (b) so that it contains as few constants with unknown values as possible.

(d) For each constant still represented by a letter in your formula from (c), determine whether it is positive or negative.

4. Suppose that p and q are polynomials such that the degree of p is greater than or equal to that of q, and that f is the rational function defined by the formula $f(x) = p(x)/q(x)$. Then p can be divided by q using polynomial long division to rewrite f in the form $f(x) = r(x) + s(x)/q(x)$, where r and s are polynomials and the degree of s is less than that of q. (If this is not obvious, try a few examples to see why this is true. If you have forgotten how to do polynomial long division, check with the other members of your group or with the tutors in the Math Lab.) Rewriting rational functions this way, using polynomial long division, can give you quite a bit of information about the behavior of f as $x \to \pm\infty$. Why? Illustrate using both graphical and numerical examples.