Math 115 Group Homework 4
Fall, 1999

1. Suppose that $f(x) = x^3$ and $g(x) = x^3 - 6x^2 + 11x$.

   (a) Are there any positive numbers x_0 such that the average rate of change of f between 0 and x_0 equals the instantaneous rate of change of f at x_0? Explain, referring only to a sketch of the graph and not to any computations you might happen to know how to do with derivatives.

   (b) Are there any positive numbers x_0 such that the average rate of change of g between 0 and x_0 equals the instantaneous rate of change of g at x_0? Explain, referring only to a sketch of the graph and not to any computations you might happen to know how to do with derivatives.

2. Set your calculator to compute a table of values for $f(x) = 3^x$ when $x = 1.7$ through 2.3 in increments of 0.1. (On the TI-83, this can be done by letting Y_1 equal 3^x as if you were going to graph this function, then going to the 2nd TBLSET menu and setting TblStart=1.7, ΔTbl=.1, making sure that Indpnt and Depend are both set to Auto, and pressing 2nd TABLE. You can read any value that appears in the table with additional accuracy by using the arrow keys to move to it and then reading the value from the bottom of the screen.)

   (a) From this table, estimate $f'(2)$.

   (b) What changes would you make to your calculator settings to get more accuracy in your estimate of $f'(2)$? Explain, making reference to the definition of the derivative, and illustrate by approximating $f'(2)$ until you are certain you have the value rounded accurately to two places after the decimal point. (A warning: If you carry this procedure to great extremes, you will discover that in your effort to get more accuracy you will suddenly lose a whole bunch of accuracy all at once. This is due to certain effects stemming from the fact that your calculator does not compute exact values, but has to round its results internally to some finite number of significant digits, usually about 13.)

3. Let $g(x) = 5 + 3^x$.

   (a) Use the procedure described in Exercise 2 to estimate $g'(2)$ to two places after the decimal point. What relationship does this have to the estimate for $f'(2)$ you obtained in Exercise 2?

   (b) By making reference to the table values you obtained for g and their relationship to those you obtained for f in Exercise 2, explain why $f'(2)$ and $g'(2)$ are related as they are.

   (c) Sketch graphs for f and g on the same axes, and explain from the graphs why you would have expected $f'(2)$ and $g'(2)$ to be related as they are.

4. Here is another calculator procedure you can use to estimate derivatives. Graph the function $f(x) = \sin(\ln(x))$ using standard zoom, use the trace key followed by the arrow keys to move your cursor to a point on the graph as close to $x = 2$ as you can get, then zoom in. Now once again use the trace key followed by the arrow keys to move your cursor to a point on the graph as close to $x = 2$ as you can get, and zoom in again. Continue this process a few times until the graph looks like a straight line. Use some procedure that involves tracing along this graph to find the slope of this straight-looking line. This slope will be an estimate for $f'(2)$. Describe the procedure you use to find the slope of the straight-looking line, and give the result of your computation. Use an argument involving tangent lines to graphs to justify this procedure for estimating derivatives.