Math 115 Group Homework 5
Fall, 1999

1. Recall the following from the uniform examination you just took:

   Sunspots are dark spots on the surface of the sun that are often larger than the earth. The number of sunspots oscillates regularly between a maximum of about 140 and a minimum of about 4, with about 11 years between successive peaks. The next maximum is predicted to occur just about at the beginning of the year 2001.

   Let $S(t)$ be the number of sunspots t years after the next peak at the beginning of the year 2001. A part of the exercise from the test involved finding a formula for $S(t)$. You will use that formula in this exercise, so make sure that your group has the correct formula.

   (a) From a calculator graph of $S(t)$ and facts you know about the shapes and symmetries of trigonometric functions, estimate when the number of sunspots is decreasing the fastest in the first decade of the next millennium. Give your answer to the nearest month. When we say that the number of sunspots is decreasing the fastest at that time, what does that mean in terms of $S'(t)$?

   (b) Estimate the value of $S'(t)$ when the number of sunspots is decreasing the fastest. Use some sort of graphical or tabular technique to do this, rather than any method you might happen to know that involves formulas, and be sure to show clearly how you obtained your estimate. This estimate should be reasonably accurate, but it will be up to you to decide what would constitute reasonable accuracy in this situation, and to defend your decision.

   (c) Trace to the point on your calculator graph where the maximum value of $S(t)$ occurs in 2001, and then zoom in on that point several times until you are looking at a greatly magnified portion of the graph around that point. Describe in as much detail as you can what this portion of the graph looks like, and explain why you would have expected to see that.

2. The Arapahoe Glacier near Boulder, Colorado tends to grow in the winter, as snow adds to its mass, and to shrink in the summer, as it melts. Let w be the total weight of the Arapahoe Glacier, and let t represent time.

   (a) Sketch a well-labeled graph of dw/dt as a function of t that represents the general behavior of dw/dt over the course of a year.

   (b) Describe weather conditions that should guarantee that $dw/dt = 0$, and explain carefully.

   (c) Suppose that a spring day in Boulder starts out with frost on the ground, but it gets hotter and hotter as the day goes along until it is sweltering in the middle of the afternoon. (Those who have lived in Boulder know that this often happens.) Would you expect dw/dt to increase or decrease during that time?

3. Just as the second derivative measures the rate of change of the first derivative, the third derivative measures the rate of change of the second derivative, while the fourth derivative measures the rate of change of the third derivative, and so forth.

   (a) Let $P(t)$ represent the average price of some standard collection of goods, so that $P'(t)$, the rate at which $P(t)$ is increasing, represents inflation. In a U.S. presidential debate some years ago, one of the candidates claimed that inflation was not nearly as much of a problem as it had been before his presidency, since "the rate at which inflation has been increasing is now growing more slowly." This is equivalent to saying that some derivative of P is now smaller than it was. Which derivative? (Incidentally, although the quote given here may not be perfectly accurate, this candidate really did say something very close to this.)

   (b) Just as the first derivative of position is velocity and the second derivative of position is acceleration, the third derivative of position is called *jerk*. Why? (HINT: If you are traveling in an auto and your velocity is constant, what do you feel? What do you feel if your velocity is not constant but your acceleration is constant? What other possibilities are there?)

4. Suppose that, on page 120 of your text, the graphs for exercises 1 through 6 represent position $f(x)$ as a function of the time x. (Yes, the letter t is usually used for time, but let's let x have a turn at it for a change.) Which of the graphs represent each of the following situations throughout the entire portion of the graph that is shown? There may be more than one graph that represents some of the given situations, so be sure to mention each one that does. If none of the graphs represents a given situation, then either sketch a graph that does or explain why no graph could.

   (a) Zero velocity.

   (b) Zero acceleration.

   (c) Zero velocity but negative acceleration.

   (d) Positive velocity and positive acceleration.

   (e) Positive velocity but negative acceleration.

   (f) Increasing velocity.