Math 115 Group Homework 6
Fall, 1999

1. Suppose that your friend Ike has read the material for this course through Section 4.1, but has not yet read 4.2 or any later sections. Unfortunately, Ike has misunderstood the power rule, and believes that $\frac{d}{dx}(e^x) = xe^{x-1}$. Your job is to convince him that he must be wrong, using only the graphs of e^x and xe^{x-1} in your arguments. Ike is hard to convince, so the more evidence you can present from the graphs, the more likely he is to be persuaded. Remember that you are not allowed to use any arguments involving differentiation formulas or numerical computations; all you can do is argue from the graphs. Of course, you may examine the graphs as closely as you like.

2. Suppose that $f(x) = e^x$ and $g(x) = ex$.

   (a) From a careful inspection of the graphs of f and g drawn on the same axes, speculate about the relationship of the graph of g to that of f. Be as specific as possible.

   (b) Show algebraically that your speculation from (a) is correct.

   (c) From what you have shown in (b) and your knowledge of properties of the graph of f, argue that $e^x \geq ex$ for all real numbers x.

3. There are some ways to take derivatives that are known as *really bad ways to differentiate,* or RBWDs for short. (Suggested pronunciation for RBWD: reeby-weedy.) These are methods that are technically correct, but much harder than necessary. One particularly outrageous example would be taking the derivative of $f(x) = 2x$ using the product rule: $f'(x) = (\frac{d}{dx}2)x + 2(\frac{d}{dx}x) = 0 \cdot x + 2 \cdot 1 = 2$.

   (a) Suppose that $g(x) = 2/x^2$. You could use the quotient rule to differentiate this, but that would be a reeby-weedy. Why? More generally, what about functions with formulas of the form $c/h(x)$, where c is a constant?

   (b) Suppose that $k(x) = \sqrt{x^5}$. You could use the formula for the derivative of the square root, along with the chain rule, to take this derivative, but that would be another reeby-weedy. Why?

   (c) Students sometimes are given the (bad) advice that they don't have to learn the quotient rule, since it is always possible to use the product rule instead as in Example 6 on page 211. This is often a reeby-weedy. Give a specific example of a quotient that can be differentiated by the product rule, but where the quotient rule is obviously much easier.

   (d) Find one other reeby-weedy not already mentioned above. Be creative!

4. Suppose that $f(x)$ and $g(x)$ are as in Figure 1.58 on page 50, and that $h(x) = f(g(x))$. Estimate $h'(0)$. As always, explain carefully. (A ruler will be very helpful.)