Math 115 Group Homework 7
Fall, 1999

1. Let E be the ellipse whose equation is $x^2 + 4y^2 = 16$.

   (a) This equation does not define y as a function of x, since for many values of x there are two corresponding values of y. However, it is still possible to use the function graphing features of your calculator to draw the graph of E. How? (Notice that it is not fair to use the drawing features of your calculator to sketch the graph as a careful piece of artwork. You must specifically use the function graphing features of the calculator to accomplish this.)

   (b) Sketch a careful graph of E, then use the graph to predict the sign of dy/dx in each quadrant.

   (c) Find a formula for dy/dx, and use it to confirm your predictions from (b).

   (d) Consider this statement: *Tangent lines on opposite sides of* E *are parallel.* This is stated a little sloppily, since it is not made completely clear what "opposite sides" means. Restate this rigorously, so that the reader cannot mistake what it means. Then use the formula from (c) to show that your statement is actually true. Mention any special points on E where the formula cannot be used, even though the result is still obviously true at those points.

2. Population biologists sometimes use a curve called the *Gompertz growth curve* to model growth in populations. The formula for the Gompertz growth curve is

   $$N = b \cdot 2^{-c \cdot 2^{-t}},$$

   where N is the size of the population, the variable t represents elapsed time and is nonnegative, and the constants b and c are positive.

   (a) Use your calculator to sketch this graph for various values of b and c, and decide what the general shape of this curve will be. Sketch a graph of this general shape by hand.

   (b) As $t \to +\infty$, what does N do? Base your argument on the equation for N, not on the appearance of the graph. (It could be very helpful to break this down into bitesized pieces by asking yourself questions like the following. As $t \to +\infty$, what does 2^{-t} do? What about $-c \cdot 2^{-t}$? What about $2^{-c \cdot 2^{-t}}$?) Use this information to locate and carefully label an asymptote on the graph you drew in (a).

   (c) Find a formula for dN/dt and simplify it as much as possible.

   (d) As $t \to +\infty$, what does dN/dt do? Use the formula you found in (c) to answer this, rather than the appearance of the graph.

   (e) Based on the appearance of the graph of N, why would you expect the result you obtained in (d)?

   (f) The Gompertz growth curve often provides a more realistic model for population growth than the usual exponential model. Why?

3. The purpose of this exercise is to show you a technique that is useful for finding the derivative of a function when it is not at all clear how to differentiate the function itself, but it *is* clear how to differentiate the logarithm of the function.

(a) Suppose that f is a differentiable function that is positive in the sense that $f(x) > 0$ for each x in the domain of f. Find a formula for $\frac{d}{dx}\ln\big(f(x)\big)$ in terms of $f(x)$ and $f'(x)$. (Notice that you are not allowed to do this just for some particular function that happens to be differentiable and positive, such as $f(x) = x^2 + 1$. You must find a general formula that works for *all* differentiable positive functions f.)

(b) Now suppose that $f(x) = x^x$, with domain $x > 0$. Notice that you cannot use the power rule to differentiate this, since the exponent is not constant, and you also cannot use the usual rules for exponential functions to differentiate this, since the base is not constant. However, it is not hard to differentiate $\ln\big(f(x)\big)$. Do so, then make appropriate substitutions into the formula you found in (a) and solve to find a formula for $f'(x)$.