Math 115 Group Homework 8
Fall, 1999

1. The purpose of this exercise is to explore the relationship between a function's zeros, local extrema, and inflection points. In the two parts of this exercise, the functions you find should have the entire real line as their domain and be differentiable everywhere.

   (a) Write down a formula for a function with an infinite number of local minima, an infinite number of local maxima, and an infinite number of inflection points, but no zeros. Of course, you must show that this function has the claimed properties.

   (b) Write down a formula for a function with exactly one zero, no critical points (and therefore no local maxima or minima), and an infinite number of inflection points. In this case, you may argue from your graph that the function has exactly one zero, but the remainder of the arguments should rely on the formulas for the function's first and second derivatives. (*Hint:* What kind of functions could you add to the function you found in part (a) that will change the first derivative but not the second derivative?)

2. Recall that if $s(t)$ is the distance an object has traveled at time t, then the third derivative of this distance function is called the *jerk* experienced by the object. That is, the jerk is $s'''(t)$ or $s^{(3)}(t)$. (The notation with the order of the derivative written as a superscript inside parentheses is occasionally used for third derivatives, and almost always used for derivatives of order four or higher, so that one does not get eyestrain trying to count large numbers of primes.)

   (a) Devise a test like the first derivative test, but using higher order derivatives of s, that would be useful for locating the local maxima and minima of the jerk. Find an appropriate name for your test.

   (b) Devise a test like the second derivative test, but using higher order derivatives of s, that would be useful for locating the local maxima and minima of the jerk. Find an appropriate name for your test.

   (c) Suppose that $s(t) = \sin(t) + 5t^3 + 3t^2 - 6t + 4$. Use the test you devised in (b) to find the smallest positive t at which the jerk derived from this position function has a local maximum.

3. Recall the family of *Gompertz growth curves* from Group Homework 7, whose members have as their formulas

$$N = b \cdot 2^{-c \cdot 2^{-t}},$$

   where N is the size of the population, the variable t represents elapsed time and is nonnegative, and the parameters b and c are positive.

   (a) What is the effect of varying the parameter b? Illustrate with sketches. What is the physical significance of this parameter? (It may help to refer to your previous group homework to answer this.)

(b) What is the effect of varying the parameter c? Illustrate with sketches. What is the physical significance of this parameter?

4. Here is an optimization problem to be solved graphically. For this, you will need to know that if a cylindrical can has height h and radius r, then its total surface area is given by the formula $A = 2pr(h + r)$. The formula for the volume of such a can is given in the list of formulas in the last few pages of your book. Suppose you are to design a cylindrical can that is to have a volume of 1000 cubic centimeters.

   (a) Find a formula for A in terms only of r rather than in terms of both r and h.

   (b) Using the graphing features of your calculator, find the value of r that will minimize the surface area A of the can. Give your answer to a large number of decimal places.

   (c) Find the value of h that your can will have when its radius is the number you found in (b). Give your answer to a large number of decimal places. Do you notice any relationship between the radius and height of this can?

   (d) If you minimize the surface area of the can by using the values of h and r you found in parts (b) and (c), what will your can look like when viewed directly from the side?

   (e) Many cans, such as old-style Green Giant vegetable cans, do have the appearance you described in (d). Other cans with the same volume, such as Campbell Soup cans, are taller and thinner. There is a small war that has gone on for years between suppliers of food in cans and the grocery stores that stock the items, with one side preferring one type of can and the other side preferring the other style. Which side do you think prefers which type of can, and why?