**Math 115 Group Homework 9**
**Fall, 1999**

1. For this exercise, it will be handy to have the guidebook for a TI-82 or TI-83 calculator handy. Consider the following entry keyed into the home screen of one of these calculators:

   `sum(seq(X^4,X,1.04,3,.04))*(1/25)`

   (The `sum` command can be found by pressing $2^{nd}$ LIST MATH, and the `seq` command by pressing $2^{nd}$ LIST OPS. It will *not* work to key the letters for these commands one at a time.)

   (a) In terms of certain sums that you have been studying lately, what does this compute? Be as specific as possible, and be sure to say why each of the numbers 1.04, .04, and 1/25 in the above expression has that particular value.

   (b) The type of sum that the above expression computes is one of two types of sums you have studied lately. Change the numbers in the expression so that it computes the *other* type of sum, again explaining why each of the numbers has the specific value you give it.

   (c) What do the sums in (a) and (b) estimate? Is the sum in (a) an underestimate or an overestimate? What about the one in (b)? Explain.

   (Thanks go to Irina Arakelian for devising the method that is the basis for this exercise, and to Karen Rhea for calling it to the exercise writer's attention.)

2. This exercise is based on the data in Table 3.1 on page 146 for the velocity of a certain car measured every two seconds. The data suggests that the velocity graph is concave downward when $0 \le t \le 10$; see Figure 3.1 on page 147. (The additional data in Table 3.2 might suggest otherwise on certain intervals during this period, but ignore that for the purposes of this exercise, and assume that the graph is indeed concave downward over the entire time interval $0 \le t \le 10$.) As mentioned in the book, the left sum of 360 feet is an underestimate of the distance traveled during this period, while the right sum of 420 feet is an overestimate. Suppose one were to average these two estimates to get a third estimate of 390 feet. Would this be an under- or overestimate of the actual distance traveled?

   HINT: In Figure 3.1, is the amount you need to add to the lower sum to get the actual area under the velocity graph more or less than half the difference between the lower and upper estimates?

3. In finding the integrals requested in this exercise, you must argue only from the geometry of the graphs, rather than from calculator computations, Riemann sums, or exact computations with formulas you might have learned elsewhere. Feel free to use any obvious symmetries that the graphs of the functions in question might have. Of course, you must justify all of your answers.

   (a) Find $\int_{-1000}^{1000} \tan^{-1}(x)\,dx$. (See page 58 for the graph of the arctangent function.)

(b) Given that $\int_0^{p} \sin(x)\,dx = 2$, find $\int_0^{np} \sin(x)\,dx$ for each positive integer $n$.

(c) Find $\int_0^{np} \cos(x)\,dx$ for each positive integer $n$.

(d) Find $\int_{-a}^{a} |x|\,dx$ for each positive real number $a$.

4. It can be shown that $\ln m = \int_1^m \frac{1}{t}\,dt$ for each positive integer $m$. (In fact, this even works if $m$ is a positive number that is *not* an integer, but that is not needed for this exercise.)

(a) By referring to Figure 3.11 on page 155 and thinking of what that picture would have looked like if $\int_1^2 \frac{1}{t}\,dt$ had been approximated using one subinterval instead of two, show that $1 > \ln 2$.

(b) By a similar argument, but using two subintervals on the interval $1 \le t \le 3$, show that $1 + \frac{1}{2} > \ln 3$.

(c) By a similar argument, show that $1 + \frac{1}{2} + \frac{1}{3} > \ln 4$.

(d) By a similar argument, show that for all positive integers $m$ greater than 3, $1 + \frac{1}{2} + \cdots + \frac{1}{m} > \ln m$.

(e) As $m \to \infty$, how large does $1 + \frac{1}{2} + \cdots + \frac{1}{m}$ get?