Math 115
Review Sheet for Exam II

This exam covers Chapters 2 and 4 and Sections 5.1 and 5.2. Although this exam is not cumulative in the usual sense, since there will be no questions whose only purpose is to test your knowledge of Chapter 1 and Appendix A, it is still the case that you must be familiar with the material from those earlier portions of the book, since much of this newer material depends upon it.

As with the review sheet for the first exam, you should not assume that a topic will not be on the exam just because it is not mentioned on this sheet. All of the material in the sections mentioned above is fair game.

General Topics and Skills

- You should understand the concept of a rate of change in its different forms, and in particular know the difference between average and instantaneous rates of change. You should be familiar with the term *derivative*, and understand that it measures the instantaneous rate of change of a function. You should be able to compute and represent rates of change algebraically (using formulas), numerically, and graphically. You should understand and be able to discuss the practical meaning and applications of a rate of change.
- You should understand the information that the signs of the first and second derivatives give you.
- You should understand the difference between an approximate answer and an exact answer, and know which methods give which.

Specific Topics and Skills by Section

2.1 You should know the difference between average and instantaneous velocity. You should be able to compute or estimate average and instantaneous velocity if you know position as a function of time. You should understand that instantaneous velocity is a limit of average velocities. You should understand the meaning of the limit notation and be able to work with it.

2.2 You should know the meaning of *derivative, tangent line,* and *instantaneous rate of change*. You should be able to estimate derivatives numerically using a table, or graphically by approximating the slope of a tangent line. You should understand how to find the equation of a tangent line if you know the derivative.

2.3 Given the graph of a function f, you should be able to sketch a graph of f'. Similarly, given the graph of a function g, you should be able to sketch a function f such that $f' = g$. You should understand how the sign of the derivative relates to whether or not the function is increasing or decreasing.

2.4 You should be able to explain in ordinary English the practical meaning of a derivative of a function that models a real-world situation. You should understand what the correct units for a derivative are. You should be familiar with the dy/dx notation for derivatives.

2.5 You should know what is meant by a second derivative, and how it is related to concavity. You should be able to explain and interpret the practical meaning of a real-world function.

4.1 Beginning with this section and continuing through Section 4.6, you are presented with a number of shortcut methods for finding derivatives, including the power rule, quotient rule, chain rule, and rules for trigonometric, exponential, and logarithmic functions (but you are not responsible for knowing the derivatives of the inverse trigonometric functions). You should be able to apply these quickly and accurately, and in particular know when each of these rules applies and when it does not. You should also be able to apply these rules to the solution of practical problems.

4.7 You should know how to find the derivative of a function that is implicitly defined, and the tangent line to the graph of such a function.

4.8 You should understand that for a differentiable function f, the tangent line at a point x_0 in the domain of f stays close to the graph of f for values of x near x_0, and therefore the tangent line approximates the graph of f near x_0. You should know how to use this to approximate $f(x)$ when x is near x_0. You should understand and be able to explain how concavity affects whether a tangent line approximation is an under- or overestimate. You should know when and how to use L'Hôpital's rule.

5.1 You should know the meaning of the terms local maximum, local minimum, critical point, and inflection point. You should know how to find each of these graphically using your calculator and algebraically using derivatives. You should understand the first and second derivative tests for local maxima and minima. You should understand what inflection points are, how to estimate their locations graphically, and how to find them using first and second derivatives. You should understand how inflection points are related to the local maxima and minima of the first derivative. You should be able to give rough sketches of functions knowing only the signs of the functions' first and second derivatives.

5.2 You should understand that functions sometimes depend on various constant parameters. It is important to understand how the functions and their graphs change if those constant parameters are changed (*e.g.*, what happens to the critical points), and it is also important to know what aspects of a family of curves stay the same even if the constants change (*e.g.*, whether all the members of the family have the same concavity).