Math 285.002
Final Exam Study Sheet

As with the two other major tests in this course, this test is closed book with no notes allowed, other than that you may have one 3×5 inch index card with anything you wish written on both sides. Calculators may not be used. All paper will be provided. If you need scratch paper, you may use the backs of the test sheets, but you cannot use any of your own paper.

The final exam will be comprehensive, although it will emphasize the material since the last major exam; approximately half of the test will be taken from that latter material. The study sheets for the previous two exams are worth looking over again. If you have lost your copies, they can be downloaded from the course web site.

The following are *some* of the most important topics and methods in the sections following 4he last exam that could be covered on this test. The fact that a topic or method is not mentioned in this study sheet does *not* mean that it will be left off the test; except where indicated otherwise, all material in all sections covered in this course is fair game. This study sheet was written before the test itself, so the material actually to be tested did not influence the topics listed. (I hope that those were enough disclaimers. Do remember Murphy's Law of Test Taking: *Any topic you fervently hope will not be covered on the exam almost certainly will.* The safest strategy is to study all of the material, and not try to guess what you can skip.)

Section 16.8. You should certainly know *how* to convert a triple integral from rectangular to cylindrical or spherical coordinates, but it is equally important to know *when* to do so, that is, the types of symmetry and shape a domain of integration has that lends itself to such conversion.

Section 16.9. Know how to accomplish changes of variables in both double and triple integrals using Jacobians. Do not make the common mistake of writing

$$\left|\frac{\partial(x,y)}{\partial(u,v)}\right| = \begin{vmatrix} \dfrac{\partial x}{\partial u} & \dfrac{\partial x}{\partial v} \\ \dfrac{\partial y}{\partial u} & \dfrac{\partial y}{\partial v} \end{vmatrix}. \qquad \text{(Wrong!)}$$

Of course, that last determinant is just $\dfrac{\partial(x,y)}{\partial(u,v)}$, not its absolute value.

Section 17.1. Know what a vector field is, and the difference between vector fields and scalar functions. Be able to treat vector fields using either of the two common notations $\mathbf{F}(x,y,z) = f(x,y,z)\,\mathbf{i} + g(x,y,z)\,\mathbf{j} + h(x,y,z)\,\mathbf{k}$ and $\mathbf{F}(x,y,z) = \langle f(x,y,z),\, g(x,y,z),\, h(x,y,z)\rangle$. Be able to match vector fields and gradient fields with their sketches, and sketch simple ones such as in the homework exercises assigned in this section. Know what it means for a vector field to be conservative, and for a scalar function to be the potential function for a vector field.

Section 17.2. Know how to find line integrals of scalar functions along piecewise-smooth curves, whether the integrals are with respect to one of the independent variables or with respect to arc length, and know how to find the line integral of a vector field along a piecewise-smooth curve with respect to arc length. Know how to write a line integral of a vector field with respect to arc length as the line integral of scalar functions with respect to the independent variables; see the bottom of page 1090. Know what it means to refer to the orientation of a curve C, and how you obtain the opposite orientation $-C$. Know how reversing the orientation of a curve affects each of the types of line integrals. Understand the relationship between line integrals of vector fields and work explained on page 1089.

Section 17.3. Know the Fundamental Theorem for Line Integrals, as well as the other theorems given in this section. Know the definitions of path independence, open set, connected set, simple curve, and simply-connected region. Know how to use the theorems of this section to determine when a vector field is conservative, and why it is so important to know when a vector field has that property. (The hint for the latter is the title of the section.) You do not have to know the material on conservation of energy for the final exam, although it is interesting to see how this gave the notion of a conservative field its name. Know how to find a potential function for a conservative vector field.

Section 17.4. Know Green's Theorem and the area formulas to which it gives rise.

Section 17.5. Know the definitions of curl and divergence and the two notations for each of them; $\operatorname{curl}\mathbf{F} = \nabla \times \mathbf{F}$ and $\operatorname{div}\mathbf{F} = \nabla \cdot \mathbf{F}$. Know the theorems of this section, especially how the curl of certain vector fields can be used to determine whether they are conservative. Know the two vector forms of Green's Theorem given on pp. 1114–1115. You do not need to know the forms of Green's identities developed in Exercises 33 and 34.

Section 17.6. Know how parametric surfaces are defined by vector-valued functions from a subset D of $\mathbb{R}^2$ to a surface S in $\mathbb{R}^3$, and be able to identify basic surfaces from their parametric representations, such as those given in the first five assigned homework exercises of this section. Know the definitions of the vectors $\mathbf{r}_u$ and $\mathbf{r}_v$ such that $\mathbf{r}_u \times \mathbf{r}_v$ forms a normal vector to the surface S at each point, and how equations of tangent planes can be obtained from that. Know what it means for a surface to be smooth, and the definition of the area of a smooth surface given at the bottom of page 1122. Know also how to compute the area of a surface given by a graph as described on pp. 1123–1124.

Section 17.7. Know the definition of a surface integral of a scalar function when the surface is either the graph of a function or given parametrically. Know what it means for a surface to be oriented, and the standard positive orientations of surfaces that are given by graphs or are closed. Know the definition of a surface integral of a vector field, and how the surface integral is related to the flux of such a function. Know how to obtain such an integral when the surface is either the graph of a function or given parametrically. You do not have to know the material on Gauss's Law, heat flow, and conductivity given at the end of this section, although you should remember having seen it here in case you run into it in a physics or engineering course. Know how to find the total mass of a thin sheet shaped like a given surface and how to find the center of mass of such a sheet, as described on page 1128.

Section 17.8. Know how the orientation of a piecewise-smooth surface bounded by a simple, closed, piecewise-smooth boundary curve gives rise to an orientation for the boundary. Know Stokes's Theorem and how it establishes a relationship between the curl of a velocity field for a fluid and the rotational motion of the fluid.

Section 17.9. Know what it means for a solid region to be simple. Know the Divergence Theorem and how it establishes a relationship between the divergence of a velocity field for a fluid and the net outward flow of a fluid near each point.

Section 17.10. Know what these pictures mean.

Material from Math 185, 186, and other mathematics courses that is of particular importance. Of course, it is not possible to make an exhaustive list of these topics within a reasonable amount of space, but here are some particular items that you may want to review: The exact values of the trig functions for the standard domain values 0, $\pi/6$, $\pi/4$, $\pi/3$, $\pi/2$, and π; how to find equations of tangent lines; the version of the Fundamental Theorem of Calculus that tells you how to take the derivative of a function defined by an integral; and the various rules and techniques for differentiation and integration.

Other items to which you should pay attention.

1. Use correct notation (particularly making certain that you write arrows over vector quantities such as $\vec{r}(t)$, since you cannot easily duplicate the boldface notion $\mathbf{r}(t)$ used in the text and on these handouts), and write clearly and organize your work neatly.
2. Do not use the method of proof that involves "reduction of an equation to one that you know is true," as is sometimes taught in trigonometry. It was shown in class how this can always be avoided.
3. When you are using a parameter and know its value, be sure to substitute that value into your final answer. For example, if your final answer is $r^2 - x^2$, but you know that $r = 3$, then you should write the answer as $9 - x^2$.
4. Watch for missing parentheses. For example, the expression $\int x^2 - x\,dx$ is incorrect; the correct expression is $\int (x^2 - x)\,dx$.
5. Do not make the mistake, which seemed to be growing more common near the end of the course, of assuming that any integral over a symmetric interval $[-a, a]$ must equal twice the integral over $[0, a]$; that is true only in very rare circumstances. Also do not assume that any integral containing trig functions over the interval $[-\pi, \pi]$ equals the integral of the same expression over $[0, 2\pi]$. That generally does not happen.
6. Points can be deducted if your work and answer are technically correct, but indicate a probable misunderstanding of a method. For example, if someone writes $\int_1^5 (1/t)\,dt = |\ln 5|$, then the answer is technically correct, but it appears more than likely that the writer misremembered the formula for $\ln t$, thought there had to be some absolute values in there somewhere, and just got lucky that $\ln 5$ happens to be positive.