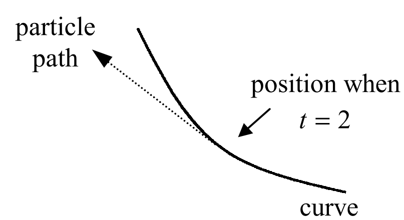
Math 285.002
Hour Exam 1
October 13, 2000

Name

This is a fifty-minute test. You may have one 3×5 card with anything you wish written on its two sides, but other than that this is a closed-book test. No calculators are allowed. Do all of your work directly on these sheets, using the backs for scratch paper if necessary but without using any of your own scratch paper. Organize your work carefully, and make certain it is clear how you obtain your results. *Circle your answers.*

1. (20 points) A particle is traveling along the curve $x = 3 - \frac{1}{2}t^2$, $y = t^3 - 1$ in the direction of increasing t, but when $t = 2$ it leaves the curve and continues traveling along a straight line path in the direction it was going at that time, which will result in its striking the y-axis. At what y-value will it strike the y-axis?



2. Suppose that $\mathbf{r}'(t) = \langle \cos t, \sin t, t \rangle$ whenever $t \in \mathbb{R}$ and that $\mathbf{r}(0) = \langle 0, 1, 2 \rangle$.

   (a) (10 points) Find a formula for $\mathbf{r}(t)$.

   (b) (10 points) Is the curve given by $\mathbf{r}$ smooth? Why or why not? Make sure your answer addresses all necessary points. (You may not argue from the appearance of the graph. You must use the definition of smoothness.)

3. Suppose a farmer has a small field whose boundary is given in polar coordinates by the formula $r = 10\theta^2$, $-\pi \le \theta \le \pi$, where r is in meters.

   (a) (4 points) Sketch the field.

   (b) (8 points) Find the area of the field, being sure to include units.

   (c) (8 points) Find the length of the boundary of this field, being sure to include units.

4. A plane P_1 has normal vector $\langle 1,2,3 \rangle$, plane P_2 has normal vector $\langle 4,-1,5 \rangle$, and the line L of intersection of P_1 and P_2 passes through the point $(6,7,-2)$.

(a) (14 points) Find symmetric equations for L.

(b) (6 points) Convert the symmetric equations from (a) into cylindrical and spherical coordinates, clearly labeling which is which.

5. Suppose that $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ are vectors in $\mathbb{R}^3$, no two of which are parallel. Let θ be the angle between $\mathbf{b}$ and $\mathbf{c}$, and let ϕ be the angle between $\mathbf{a}$ and $\mathbf{b}\times\mathbf{c}$. Let V be the volume of the parallelepiped determined by $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$.

   You will want to sketch this, but do not be fooled by your sketch into believing that $0 \le \phi \le \frac{\pi}{2}$. If $\mathbf{a}$ and $\mathbf{b}\times\mathbf{c}$ lie on opposite sides of the plane determined by $\mathbf{b}$ and $\mathbf{c}$, then $\frac{\pi}{2} \le \phi \le \pi$. In general, you may only assume that $0 \le \theta \le \pi$ and that $0 \le \phi \le \pi$.

   (a) (10 points) Beginning with the formula for V given in this course, find a formula for V containing $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$, θ, and ϕ, but with no dot or cross products.

   (b) (10 points) From the formula obtained in (a), argue clearly and in complete sentences that a parallelepiped with three adjacent sides of length $|\mathbf{a}|$, $|\mathbf{b}|$, and $|\mathbf{c}|$ has its largest volume if the three sides are mutually perpendicular.