Math 285.002
Hour Exam 1 Study Sheet

The general rules for the hour exams and final exam are that the tests are closed book with no notes allowed, other than that you may have one 3×5 inch index card with anything you wish written on both sides, and that calculators may not be used. All paper will be provided. If you need scratch paper, you may use the backs of the test sheets, but you cannot use any of your own paper.

The following are *some* of the most important topics and methods in each section that could be covered on this test. The fact that a topic or method is not mentioned in this study sheet does *not* mean that it will be left off the test; except where indicated otherwise, all material in Chapters 11 and 13 and Sections 14.1 and 14.2 is fair game. This study sheet was written before the test itself, so the material actually to be tested did not influence the topics listed. (I hope that those were enough disclaimers. Do remember Murphy's Law of Test Taking: *Any topic you fervently hope will not be covered on the exam almost certainly will.* The safest strategy is to study all of the material, and not try to guess what you can skip.)

Section 11.1. Know how curves in $\mathbb{R}^2$ are defined by parametric equations, and what their initial and terminal points are. You do not need to know the definitions of the other special types of parametric curves given in this section, such as Bézier curves, cycloids, and conchoids, but you should be able to recognize simple parametrized curves, such as circles and ellipses, from the equations defining them.

Section 11.2. Know how to find slopes and equations of tangent lines to parametrized curves in $\mathbb{R}^2$. Know also how to find second derivatives. For this test, it is not necessary to know how to do a detailed analysis of parameter intervals as in Example 2 on pp. 684–5, but that topic could still be fair game for the comprehensive final exam. Know how to find areas under parametrized curves as on pp. 685–6. You may ignore the material on graphing calculators and computers on pp. 686–7.

Section 11.3. Be able to find the lengths of parametrized curves in $\mathbb{R}^2$ by the methods of this section, and the areas of the surfaces formed when they are rotated about either axis.

Section 11.4. Be able to convert between polar to Cartesian coordinates, and know how to plot simple polar curves, particularly using the "auxiliary equation" method give in class in which a polar graph is deduced from a corresponding Cartesian graph. Know how to find the Cartesian equation of a tangent line to a polar graph at a given point. You may ignore the material on graphing polar curves with graphing devices given on pp. 701–2.

Section 11.5. Know how to find the area enclosed by a polar graph and the length of such a graph as described in this section. Be cautious about using the correct starting and ending θ values when doing this, particularly to avoid retracing a polar curve and thus computing too large a number for arc length or area.

Section 11.6. Know the standard equations of the conic sections given in this section, as well as how to find all parameters in those equations as well as (where appropriate) vertices, foci, major axes, and equations of directrices and asymptotes. Be able to deal with shifted conics.

Section 11.7. Know how the equations $r=\dfrac{ed}{1\pm e\cos\theta}$ and $r=\dfrac{ed}{1\pm e\sin\theta}$ define conic sections in polar coordinates, and the meaning of the numbers e and d (and do recall that, by the book's development, the number d must be positive). Know how to recognize from the equation which type of conic section is represented, and its orientation.

Section 13.1. Know how to plot points in $\mathbb{R}^3$ using Cartesian coordinates, and how to recognize simple surfaces in $\mathbb{R}^3$, such as planes and spheres, from their equations. Know the distance formula for $\mathbb{R}^3$.

Section 13.2. A general comment about vectors: Much of what we have studied about vectors works in both $\mathbb{R}^2$ and $\mathbb{R}^3$, even if our development was done for only one of these types of vectors. Know when facts we have learned about vectors apply to both of these situations, and when to only one of them. In this particular section, know the definition of a vector, what a representation of a vector is, how a position vector differs from a general representation of a vector, and all of the introductory definitions and facts about vectors presented in this section.

Section 13.3. Know the general properties of the dot product and the important theorem that $\mathbf{a}\cdot\mathbf{b}=|\mathbf{a}||\mathbf{b}|\cos\theta$, where θ is the angle between the vectors $\mathbf{a}$ and $\mathbf{b}$. Know how this last fact can be used to find the angle between two vectors, and the resulting characterization of when two vectors are orthogonal. Know how to find direction angles and direction cosines and what they represent. Be able to find scalar and vector projections and use them in applied problems.

Section 13.4. Be able to compute cross products, and know the two fundamental theorems about cross products on p. 839 as well as the corollary on p. 840. Know how to use cross products (and scalar triple products) to find areas of parallelograms and volumes of parallelepipeds. Know items 1–5 in Theorem 8 on p. 841 (which are mostly facts you would expect to hold, except for the somewhat surprising and very important item 1).

Section 13.5. Know the various standard equations of lines and planes. Know how to do all of the following (over each of which you had homework exercises): find an equation for a line through a given point parallel to a given vector; find an equation for a line through a given point perpendicular to a given plane; find an equation for a line through two points; find an equation for a line through a given point parallel to a given line; determine whether two given lines are parallel, skew, or intersecting; find the equation of a plane through a given point perpendicular to a given vector; find the equation of a plane through a given point parallel to a given plane; find the equation of a plane through three given points; find the equation of a plane through a given point containing a given line; find the point at which a given line intersects a given plane; find the equation of the line of intersection of two planes; and determine whether two planes are parallel, perpendicular, or neither, and if neither, then find the angle between them. This list is not exhaustive. Lines and planes are an extremely important topic in this course and other courses that depend on this one, so you should be very familiar with how to work with them.

Section 13.6. Know the definition of a cylinder and a quadric surface, and the formulas in the table on p. 858 that determine the types of certain quadric surfaces.

Section 13.7. Know how spherical and cylindrical coordinate are defined, how to convert between the three standard types of coordinates in $\mathbb{R}^3$ (including Cartesian), and the types of figures whose graphing is facilitated by spherical and cylindrical coordinates.

Section 14.1. Know how vector functions and their limits are defined. Be aware of the relationship between vector functions, space curves, and parametric equations. You are not responsible for the material on using computers to draw space curves on pp. 874–5.

Section 14.2. Know how derivatives and integrals of space functions are defined and computed. Know the definition of a tangent vector, the tangent line, and the unit tangent vector to a space curve at a point. Know what it means for a curve to be smooth, and the basic differentiation rules on p. 880 in Theorem 3.

Material from Math 185, 186, and other mathematics courses that is of particular importance. Of course, it is not possible to make an exhaustive list of these topics within a reasonable amount of space, but here are some particular items that you may want to review: The exact values of the trig functions for the standard domain values 0, $\pi/6$, $\pi/4$, $\pi/3$, $\pi/2$, and π; how to find equations of tangent lines; the version of the Fundamental Theorem of Calculus that tells you how to take the derivative of a function defined by an integral; and the various rules and techniques for differentiation and integration.

Other items to which you should pay attention.

1. Use correct notation, and write clearly and organize your work neatly.
2. Do not use the method of proof that involves "reduction of an equation to one that you know is true," as is sometimes taught in trigonometry. It was shown in class how this can always be avoided.
3. When you are using a parameter and know its value, be sure to substitute that value into your final answer. For example, if your final answer is $r^2 - x^2$, but you know that $r = 3$, then you should write the answer as $9 - x^2$.
4. Watch for missing parentheses. For example, the expression $\int x^2 - x\,dx$ is incorrect; the correct expression is $\int (x^2 - x)\,dx$.
5. Points can be deducted if your work and answer are technically correct, but indicate a probable misunderstanding of a method. For example, if someone writes $\int_1^5 (1/t)\,dt = |\ln 5|$, then the answer is technically correct, but it appears more than likely that the writer misremembered the formula for $\ln t$, thought there had to be some absolute values in there somewhere, and just got lucky that $\ln 5$ happens to be positive.