Math 285.002
Hour Exam 2
November 17, 2000

Name _____

This is a fifty-minute test. You may have one 3×5 card with anything you wish written on its two sides, but other than that this is a closed-book test. No calculators are allowed. Do all of your work directly on these sheets, using the backs for scratch paper if necessary but without using any of your own scratch paper. Organize your work carefully, and make certain it is clear how you obtain your results. *Circle your answers, and simplify them as much as possible.*

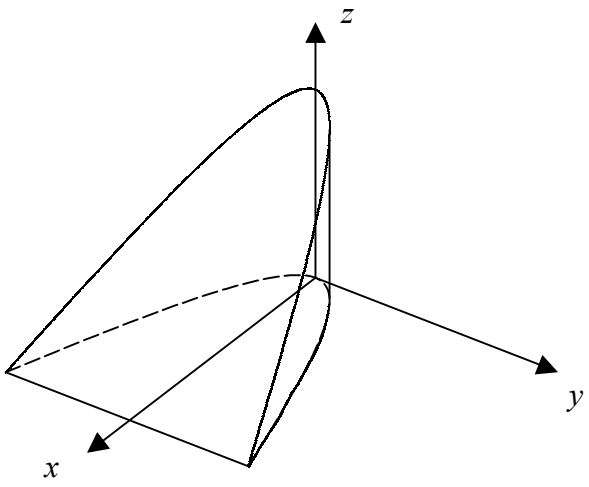
1. A particle is moving so that its position vector at time t is $\mathbf{r}(t) = \frac{3}{2}t^2\mathbf{i} + \frac{4}{3}t^3\mathbf{j}$.

   (a) (10 points) Find the curvature when $t = 1$.

   (b) (10 points) Find the tangential and normal components of the acceleration vector when $t = 1$. Be sure to label which is which.

2. (20 points) Let $f(x,y) = 6xy^2 - 2x^3 - 3y^4$. Find the critical points of this function and classify each as being a local maximum, local minimum, or saddle point. Make sure your work can be followed easily for *each* of the critical points.

3. (20 points) Let E be the region bounded by the parabolic cylinder $x = y^2$ and the planes $z = 0$ and $x + z = 1$; see the illustration. Find the volume of E.



4. (20 points) A lamina consists of the portion of the disk of radius 4 centered at the origin that lies in the first quadrant, and its density is proportional to the distance from the origin. Find the center of mass of the lamina.

5. Suppose that the tangent plane to the level surface $F(x,y,z)=k$ at the point $(4,0,-2)$ is given by the equation $4x-6y+12z-45=0$, and that $F_x(4,0,-2)=2$.

   (a) (10 points) Find $\nabla F(4,0,-2)$.

   (b) (10 points) Suppose that you begin at the point $(4,0,-2)$ and move 21 units in the direction in which F is increasing the most rapidly at the point $(4,0,-2)$. At what point do you end up? (If for some reason this depends on part (a) and you cannot find the answer to (a), then use the [incorrect] value $\langle 1,2,3\rangle$ for $\nabla F(4,0,-2)$, but be aware that if you choose this option then you cannot get full credit for this exercise.)