Math 285.002
Hour Exam 2 Study Sheet

As with Hour Exam 1, this test is closed book with no notes allowed, other than that you may have one 3×5 inch index card with anything you wish written on both sides. Calculators may not be used. All paper will be provided. If you need scratch paper, you may use the backs of the test sheets, but you cannot use any of your own paper.

The following are *some* of the most important topics and methods in each section that could be covered on this test. The fact that a topic or method is not mentioned in this study sheet does *not* mean that it will be left off the test; except where indicated otherwise, all material in Sections 14.3 through 16.7 is fair game. This study sheet was written before the test itself, so the material actually to be tested did not influence the topics listed. (I hope that those were enough disclaimers. Do remember Murphy's Law of Test Taking: *Any topic you fervently hope will not be covered on the exam almost certainly will.* The safest strategy is to study all of the material, and not try to guess what you can skip.)

Section 14.3. Know the various forms of the formulas for arc length and curvature, as well as how to parametrize a curve with respect to arc length. (You do not have to know the formula given in Exercise 30 in this section, although you did use it in a few homework exercises.) Know the relationship between the unit tangent vector $\mathbf{T}(t)$ to a smooth space curve, the principal unit normal vector $\mathbf{N}(t)$ to the curve, and the binormal vector $\mathbf{B}(t)$ to the curve. Know how to find the equations of the normal and osculating planes to a smooth space curve.

Section 14.4. Given a position vector $\mathbf{r}(t)$, know how to find the velocity and acceleration vectors $\mathbf{v}(t)$ and $\mathbf{a}(t)$, as well as the speed $v(t)=|\mathbf{v}(t)|$. Know Newton's Second Law of Motion, and how to find the tangential and normal components of an acceleration vector. You do not need to know Kepler's Laws, although do remember that you saw them here just in case you need them for a physics or astronomy course.

Section 15.1. Know the definition of a real-valued function of several variables, as well as the notions of domain, range, graph, and independent and dependent variables as they apply to such functions. Know how to find and sketch level curves and contour maps for functions of two variables, and level surfaces for functions of three variables.

Section 15.2. Know the definition of limit of a function of two or more variables, as given on pages 922 and 928 of the text, and the definitions of continuity given on pages 925 and 928, as well as what those definitions mean in common terms. Know how to apply the fact about discontinuous functions given in the red box on page 923. Know what a polynomial of two or more variables is.

Section 15.3. Know the definition of a partial derivative of first or higher order, and the various notations for partial derivatives, such as $\frac{\partial y}{\partial x}$, $\frac{\partial^3 f}{\partial x\,\partial z\,\partial y}$, and f_{xy}. Be sure to understand the order of differentiation implied by each of the notations for partial derivatives of second or higher order, and the conditions under which Clairaut's Theorem says that the order makes no difference. Know Laplace's equation, and be aware that only harmonic functions satisfy it; that is the actual definition of a harmonic function. You do not have to know the wave equation or the Cobb-Douglas production function. Know how to do implicit partial differentiation.

Section 15.4. Be able to find the equation of the tangent plane to a surface S with equation $z = f(x, y)$. Know how tangent planes are used to find linear approximations, and how linear approximations can also be found for functions of three or more variables. Know the definition of differentiability for a function of two variables, as well as the theorem on page 946 that tells when differentiability is assured. Know how to find increments and differentials for functions of two or more variables.

Section 15.5. Know and be able to apply the various forms of the chain rule given in this section. Know the formulas for implicit differentiation given on pp. 956–957 that follow from different versions of the Implicit Function Theorem. Be able to do applied problems using this material, such as assigned homework exercises 35 and 37 from this section.

Section 15.6. Understand the definition and significance of directional derivatives and the gradient vector, and know the various formulas used to compute these objects. Know how the directional derivative is used to find the direction in which a differentiable function of two or three variables is increasing most rapidly, and the value of that largest rate of change. Be able to find equations of tangent planes and normal lines to level surfaces of functions of three variables using gradient vectors. You do not need to have memorized the facts about gradient vectors given in assigned exercise 35 from this section.

Section 15.7. Know the definition of a local maximum, local minimum, and saddle point, as well as those of an absolute maximum and minimum. Know the definition of a critical point of a function of two variables, and the theorem on page 973 relating local extrema and critical points. Know the Second Derivative Test, including the extension given in class (that if $D = 0$, then the test gives no information). Know the Extreme Value Theorem for Functions of Two Variables and its application to finding absolute maxima and minima on a closed bounded set.

Section 15.8. Know how to use the method of Lagrange multipliers to find the absolute maximum and minimum values of a function subject to one or two constraints.

Section 16.1. Know how the double integral of a function of two variables is defined using double Riemann sums, and the relationship to volume for a nonnegative function. Know the Midpoint Rule for Double Integrals and the formula for average value in terms of a double integral. Know the properties of double integrals given on page 1008.

Section 16.2. Know the definition of an iterated integral of a function of two variables, and how Fubini's theorem relates double and iterated integrals.

Section 16.3. Know how to compute double integrals over type I and type II regions, or over regions that can be decomposed into pieces that are type I or II. Being able to find the lower and upper limits of integration for such regions is sometimes a bit of an art, so be sure you have a solid grasp of this art for the assigned homework exercises. Know the properties of double integrals given on pp. 1020–1021. Know how to reverse the order of integration when integrating over a region that is both type I and II; see, for example, assigned homework exercises 35 and 39 from this section.

Section 16.4. Know how to change to polar coordinates in a double integral when integrating over a polar rectangle or a region that is of the form given in the red box at the bottom of page 1026. More generally, be able to recognize *when* it is of value to change to polar coordinates to evaluate a double integral, in case the instructions for an exercise do not explicitly tell you to make such a change.

Section 16.5. Given a lamina and a mass density function for the lamina, be able to compute all of the following quantities for the lamina: the mass, the moments about the x- and y-axes, the center of mass, the moments of inertia about the x- and y-axes and origin, and the radii of gyration with respect to the x- and y-axes. Know also how to use double integrals to compute the total amount of a quantity, given a density function for the quantity, as in Example 1 on page 1030 and assigned exercise 1 from this section. Know how to use the joint probability density function (joint p.d.f.) of two continuous-type random variables to compute probabilities and expected values.

Section 16.6. Be able to use the formulas from this section to find the area of a surface given by the graph of a function of two variables.

Section 16.7. Know how triple integrals are obtained from triple Riemann sums, and how Fubini's Theorem for Triple Integrals relates triple and iterated integrals. Know also how to compute triple integrals over more general regions than rectangular solids, as discussed on pages 1044–1047. Know how triple integrals can be used to compute volumes, and how, given a mass density function for a solid, triple integrals are used to compute the solid's mass, moments about the coordinate planes, center of mass, and moments of inertia about the coordinate axes. Given three continuous-type random variables with a joint probability distribution, be able to use the joint probability density function (joint p.d.f.) of the random variables to compute probabilities. Know how to change the order of integration in a triple integral, as you practiced in assigned homework exercises 25 and 31 from this section. Know the important average value formula you used in assigned exercise 45.

Material from Math 185, 186, and other mathematics courses that is of particular importance. Of course, it is not possible to make an exhaustive list of these topics within a reasonable amount of space, but here are some particular items that you may want to review: The exact values of the trig functions for the standard domain values 0, $\pi/6$, $\pi/4$, $\pi/3$, $\pi/2$, and π; how to find equations of tangent lines; the version of the Fundamental Theorem of Calculus that tells you how to take the derivative of a function defined by an integral; and the various rules and techniques for differentiation and integration.

Other items to which you should pay attention.

1. Use correct notation (particularly making certain that you write arrows over vector quantities such as $\vec{r}(t)$, since you cannot easily duplicate the boldface notion $\mathbf{r}(t)$ used in the text and on these handouts), and write clearly and organize your work neatly.
2. Do not use the method of proof that involves "reduction of an equation to one that you know is true," as is sometimes taught in trigonometry. It was shown in class how this can always be avoided.
3. When you are using a parameter and know its value, be sure to substitute that value into your final answer. For example, if your final answer is $r^2 - x^2$, but you know that $r = 3$, then you should write the answer as $9 - x^2$.
4. Watch for missing parentheses. For example, the expression $\int x^2 - x\, dx$ is incorrect; the correct expression is $\int (x^2 - x)\, dx$.

5. Points can be deducted if your work and answer are technically correct, but indicate a probable misunderstanding of a method. For example, if someone writes $\int_1^5 (1/t)\, dt = |\ln 5|$, then the answer is technically correct, but it appears more than likely that the writer misremembered the formula for $\ln t$, thought there had to be some absolute values in there somewhere, and just got lucky that $\ln 5$ happens to be positive.