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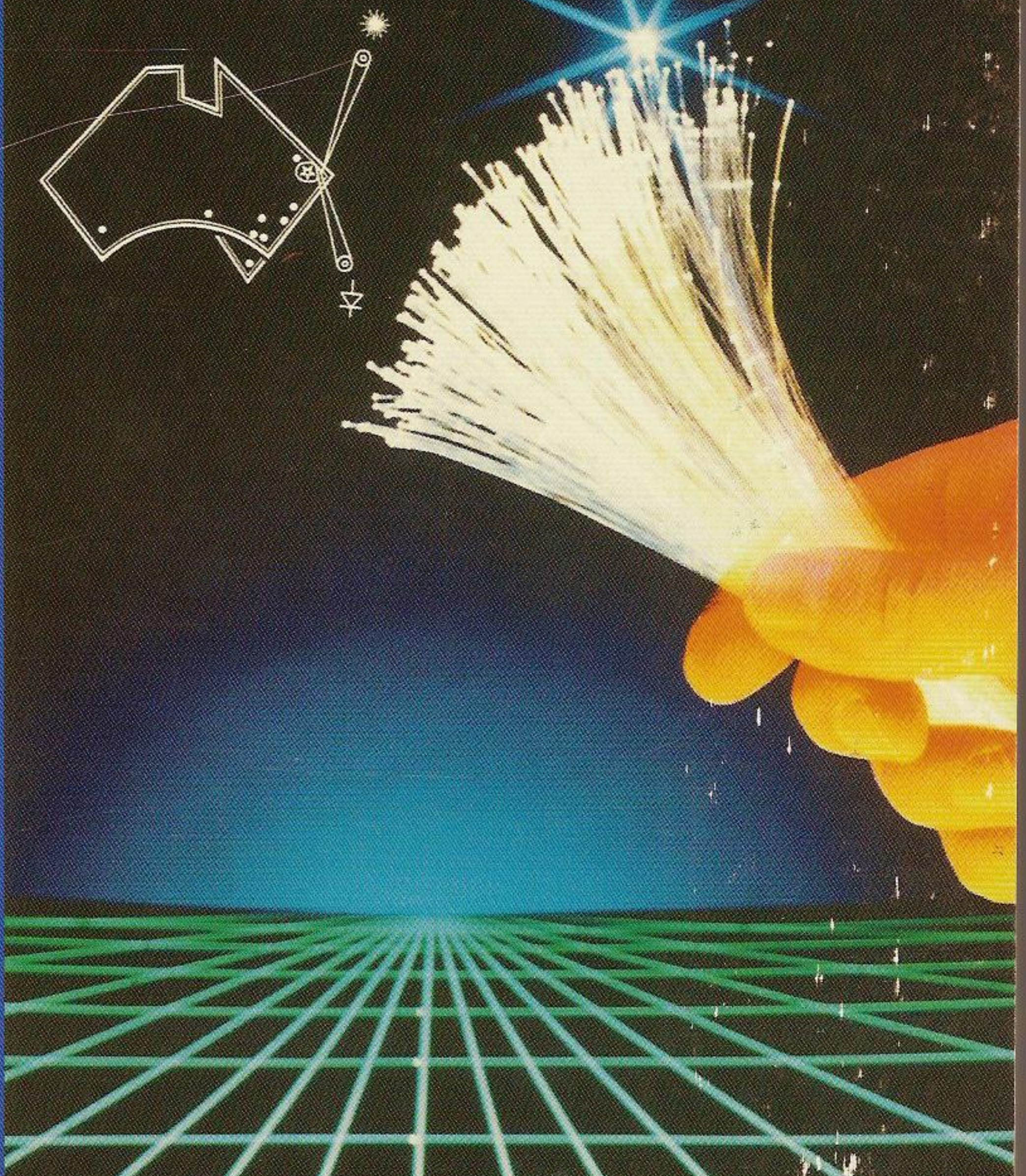

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X-junctions without Cross-Talk

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The transmission properties of certain three-dimensional waveguide X-junctions are considered. Very simple exact calculations show that for a large class of such junctions modeled by the scalar wave equation, transversely bound waves incident in one channel can pass through the X-junction without any splitting into the intersecting channel or emission of radiation. Reflection can also be completely eliminated within each channel. The devices we consider lend themselves to particularly simple fabrication by ultraviolet exposure techniques.

1 Introduction

Several recent investigations of waveguides modeled by the linear Schrödinger equation have shown that an X-junction waveguide whose refractive index profile matches that self-induced in a self-defocusing Kerr medium by a collision of two dark solitons has a number of unusual and useful properties [1, 2, 3, 4]. Firstly, such a device is lossless. This means that waves incident on the junction that are initially bound within one of the arms always emerge bound within the output arms, with absolutely no loss to radiation at the junction. Secondly, there is some freedom in coordinate transformations that allow the device to be scaled to a range of sizes, limited only by the available variation in refractive index. But above all, the most important property of these devices is that bound waves incident in one of the channels pass through the junction without any transfer of power into the intersecting channel. This, along with the lossless property, means that all of the power input is collected in the corresponding output channel, as if the channels did not intersect at all. We call this the *zero-crosstalk* property, because such devices could be used in a single-level integrated optical circuits to allow signal transmission channels to intersect physically without intersecting logically.

A valid criticism of the theory of these devices, whose refractive index profiles are based on the mathematical theory of dark solitons, is that they may be difficult to fabricate with existing technology. Current fabrication methods are best at producing devices with step-index profiles, and the refractive index profile of a device based on dark solitons as in Figure 1 (left) is clearly not in this category. So, the challenge is to determine whether there

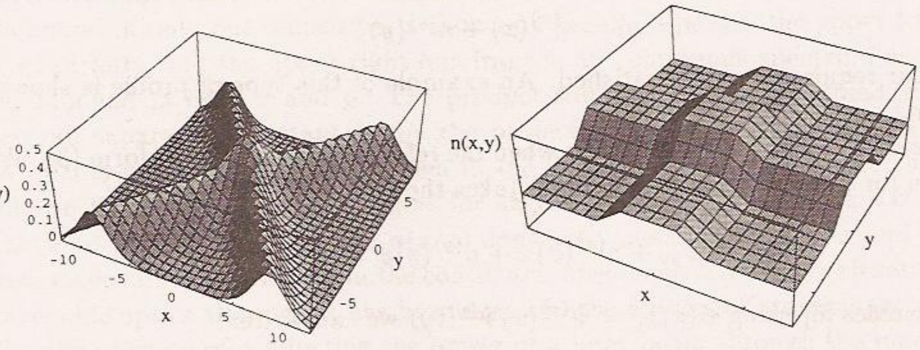

Figure 1: *Left: The refractive index self-induced in a defocusing Kerr medium by a collision of dark solitons. Right: A separable refractive index profile corresponding to a waveguide junction.*

exist junction waveguides with the zero-crosstalk property that are not connected with dark solitons at all, and that might have step-index profiles.

A glance at the zero-crosstalk X-junction whose refractive index profile is shown in Figure 1 (left) reveals one feature in particular that might be of some heuristic use — the refractive index is higher in the intersection region of the channels than in the channels themselves. Such a structure evidently provides a lensing effect to counteract the diffraction of a beam as it enters the junction. In the absence of this lensing, the beam spreads as it enters the junction and does not excite the corresponding mode on the other side of the junction with all of the input power [5]. Our intuition thus suggests that a zero-crosstalk X-junction should have an enhanced refractive index in the collision region.

In this paper we show that this intuition provides the clue to the construction of a large class of X-junction waveguides that have the zero-crosstalk property, and that are not connected with the mathematics of dark solitons. Our elementary analysis is at the level of the scalar wave equation, a more fundamental model than the Schrödinger equation. The class of X-junctions we consider includes many that are well-approximated by step-index profiles.

2 Theory

Let us start with three-dimensional scalar wave equation

$$E_{xx} + E_{yy} + E_{zz} + \frac{n(x,y)}{c^2} E_{tt} = 0 \,. \tag{1}$$

Consider monochromatic waves of the form

$$E = \psi \, \exp[ik_0\beta z - i\omega t]$$

where $k_0 = \omega/c$. In the normalized transverse variables $x \to k_0 x$, $y \to k_0 y$, the wave equation takes the form

$$\psi_{xx} + \psi_{yy} + n(x,y)\psi = \beta^2 \psi \,. \tag{2}$$

Now, we consider how to specify the refractive index $n(x,y)$ in such a way as to obtain a junction waveguide with the zero-crosstalk property. There are two essential properties that we must incorporate into $n(x,y)$. First, in the regions away from the junction it should consist of two channels, each capable of carrying at least one mode. To be specific, we assume the channels to be at right angles to each other, and then without loss of generality, we assume that the channels are parallel to the x and y axes. Second, according to our intuition, the junction region should have a refractive index that is higher than that in either of the channels. Observe that if the refractive index profile $n(x,y)$ is separable and has the form

$$n(x,y) = n^{(x)}(x) + n^{(y)}(y) \,, \tag{3}$$

then both of our requirements are satisfied. An example of this type of profile is shown in Figure 1 (right).

Let us analyze the scalar wave equation when the refractive index has the form (3). With this expression for $n(x,y)$, the wave equation takes the form

$$\psi_{xx} + \psi_{yy} + n^{(x)}(x)\psi + n^{(y)}(y)\psi = \beta^2 \psi \,. \tag{4}$$

Separating variables [6] using $\psi(x,y) = \Psi^{(x)}(x)\Psi^{(y)}(y)$ we can write:

$$\Psi^{(x)\prime\prime} + n^{(x)}\Psi^{(x)} = \lambda^{(x)2}\Psi^{(x)} \,, \tag{5}$$

$$\Psi^{(y)\prime\prime} + n^{(y)}\Psi^{(y)} = \lambda^{(y)2}\Psi^{(y)} \,, \tag{6}$$

where the separation constants $\lambda^{(x)}$ and $\lambda^{(y)}$ satisfy the relation

$$\lambda^{(x)2} + \lambda^{(y)2} = \beta^2 , \tag{7}$$

for a fixed value of the propagation constant β in the direction perpendicular to the slab. If the slab waveguide is clad, then the values of β may be quantized. Taking the channel profiles $n^{(x)}(x)$ and $n^{(y)}(y)$ each in the form of a single potential well with asymptotic values on the left and right equal to zero, we find that the individual spectra of (5) and (6) consist of the negative discrete eigenvalues $\lambda_j^{(x)2}$ and $\lambda_j^{(y)2}$ respectively, as well as the band of continuous spectrum on the positive real axis. These features are illustrated for some choice

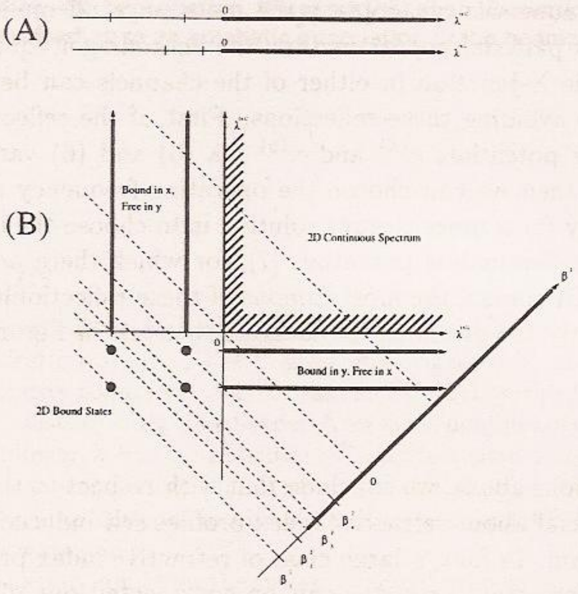

Figure 2: *The spectrum of the wave equation for an X-junction with the zero crosstalk property. (A) The real spectra of the separated one-dimensional problems in the x and y directions. (B) The corresponding spectral plane and its projection onto the real spectrum of the fully two-dimensional problem by the relation $\beta^2 = \lambda^{(x)2} + \lambda^{(y)2}$. See the text for a detailed explanation.*

of potentials $n^{(x)}$ and $n^{(y)}$ in Figure 2 (A), where discrete eigenvalues are indicated for each potential on the negative real axis, and the continuous spectrum on the positive real axis is indicated with a thick line. Now, when the product states are assembled from the solutions of (5) and (6), one can indicate the allowed solutions in the real plane with coordinates $\lambda^{(x)2}$ and $\lambda^{(y)2}$. This plane is shown in Figure 2 (B). Here, two-dimensional bound states are indicated by points in the lower-left quadrant, continuous spectrum associated with states bound in only one dimension is shown on the dark lines in the upper-left and lower-right quadrants, and the upper-right quadrant is all continuous spectrum associated with states unbound in both x and y. The product solutions can be identified with values of the overall separation constant β^2 by the projection given by $\beta^2 = \lambda^{(x)2} + \lambda^{(y)2}$. As a result of this projection, as can be seen in the figure, the continuous spectrum (indicated in bold on the β axis) generally begins for negative values of β^2 (this is a consequence of the fact that the potential function $n(x, y)$ does not vanish for all large x and y), and some discrete eigenvalues can lie within the continuous spectrum. This latter situation is unusual in waveguide optics although it has been noted in the physics of atomic spectra.

For the purpose of conducting the power of a light beam through the junction without losses, we need only to consider the product states from the continuous spectrum that are located in the upper-left and lower-right quadrants of the plane in Figure 2 (B). These states are bound in one space dimension but are free to propagate in the other, and thus represent the ideal transmission of power through the junction. Their existence establishes that waveguide junctions of the simple type considered above indeed have the zero-crosstalk property.

There remain two important questions to address. The first is the issue of whether these half-bound/half-free product solutions can be realized in a physical slab waveguide. The nature of the cladding becomes important, as it can serve to specify the available values of the overall separation constant β. However, from Figure 2 (B) it is easy to see that as long as β is sufficiently large, the existence of zero-crosstalk product solutions can be guaranteed, and this existence is then insensitive to details of the cladding.

The next question is whether it is possible to avoid back-reflections within each channel. Indeed, one of the consequences of adopting the more physical model of the scalar wave equation rather than its paraxial approximation, the Schrödinger equation, is that in principle light incident on the X-junction in either of the channels can be back-reflected. There are two approaches to avoiding these reflections. First, if the reflection coefficients $R^{(x,y)}(\lambda)$ associated with the potentials $n^{(x)}$ and $n^{(y)}$ via (5) and (6) vanish for some λ in the continuous spectrum, then we can choose the operating frequency of the device to avoid reflections. However, by far a more elegant solution is to choose the individual index profiles $n^{(x)}$ and $n^{(y)}$ to be reflectionless potentials [7], for which there are no reflections at any operating frequency. Of course, the most famous of these reflectionless potentials is $n(x) = \text{sech}^2 x$, which is exactly the profile of the isolated channels in Figure 1 (left).

3 Conclusions

Based on the simple calculations above, we conclude that with respect to the zero-crosstalk property there is nothing special about refractive index profiles self-induced by collisions of dark solitons in a Kerr medium. In fact, a large class of refractive index profiles for planar waveguides having the zero-crosstalk property can be constructed out of two individual profiles, and moreover this can be done at the level of the scalar wave equation as opposed to the Schrödinger equation, which only holds in the paraxial approximation.

It should be easy to fabricate such a waveguide junction by standard techniques, for example by photolithography or ultraviolet exposure. This is because these techniques could be used to write a single channel first, and then the intersecting channel can simply be written on top of the first, at a right angle. If the exposure technique produces a refractive index change that is roughly linear with exposure dose, then the resulting index pattern of the junction will be precisely of the type considered in the above paragraphs, with the refractive index in the collision region being the sum of the values of the refractive index in each of the channels considered separately.

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References

[1] B. Luther-Davies and Y. Xiaoping, *Opt. Lett.*, **17**, (1992).

[2] X. Yang, B. Luther-Davies and W. Królikowski, *Int. J. Nonlinear Opt. Phys.*, **2**, No 3, (1993).

[3] N. N. Akhmediev and A. Ankiewicz, *Opt. Communs.*, **100**, (1993).

[4] P. D. Miller, *Phys. Rev. E* **76**, No 4, (1996).

[5] F. J. Ladouceur and J. D. Love, *Opt. Quant. Electron.*, **24**, (1992).

[6] A. W. Snyder and J. D. Love, *Optical Waveguide Theory*, Chapman and Hall, London, 1983, Chapter 16.

[7] I. Kay and H. E. Moses, *J. Appl. Phys.*, **27**, No 12, (1956).