Effects of surface roughness on gratings

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The correlation length for sidewall roughness in planar waveguides is comparable to the period of Bragg gratings written in such structures and thus might influence the performance and the spectral properties of such gratings. Using a coupled-mode formalism, we calculate the effect of roughness or inhomogeneity for an *arbitrary* grating and present specific results for uniform and phase-shifted gratings. The broad spectral characteristics of most gratings are insensitive to roughness. However, narrow spectral features (such as transmission resonances) that rely on interference effects are affected by the presence of roughness. © 1997 Optical Society of America [S0740-3224(97)00606-1]

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1. INTRODUCTION

Roughness in most types of planar optical waveguides is due to random deviations of the waveguide sidewalls from perfect uniformity (see Fig. 1) and is caused by the deposition/etching process used during fabrication.[1] The standard deviation of these nonuniformities is typically ~1% of the waveguide width. In a previous paper[2] we showed that the effects of surface roughness in *uniform* waveguides can be neglected in most circumstances. However, the measured correlation length of the surface roughness is of the order of a few tenths of microns, which is comparable to the spatial period of reflection Bragg gratings operating at communication wavelengths (see Fig. 1). Therefore we might expect the roughness to influence the coupling properties of these gratings.

In this paper we investigate the possible impact of the surface roughness on Bragg gratings. We use a perturbative analysis of the coupled-mode equations linking the forward- and backward-traveling modes of the grating. Although we concentrate on surface roughness, the same approach can deal with bulk inhomogeneities.

2. SIDEWALL ROUGHNESS AND MODE COUPLING

We assume that the sidewall roughness can be described by local deviations of the waveguide sidewalls from perfect linearity. This is represented mathematically by a random function $f(z)$, with zero average $\langle f(z)\rangle = 0$ and standard deviation $\delta_f = [\langle f^2(z)\rangle]^{1/2}$, where z is the distance along the grating and $\langle\ \rangle$ indicates an ensemble average. Typically, for silica-based technology, $\delta_f \sim 0.05–0.1\ \mu\text{m}$. The roughness $f(z)$ possesses a well-defined autocorrelation function

$$C(z - z') = \langle f(z)f(z')\rangle. \tag{1}$$

Roughness correlations have been measured experimentally[1] and are well represented by an *exponential model* of the form

$$C(z) = \delta_f{}^2 \exp(-|z|/L_c), \tag{2}$$

where L_c is the correlation length. L_c is a measure of the correlation existing in the random signal $f(z)$ and, for silica-based technology, is of the order of 0.5 μm and is thus comparable to the period of commonly used reflection gratings.

The coupling coefficient $K(z)$ between forward and backward modes that is due to roughness can be calculated for arbitrary waveguides[3] and is proportional to the roughness function $f(z)$. As an example, and in order to quantify the analysis of later sections, we give an explicit expression for $K(z)$ for a step-index slab waveguide in Appendix A. We define σ to be the standard deviation of $K(z)$ and write $K(z) = \sigma s(z)$, where $s(z)$ possesses a normalized correlation function given by

$$\Gamma(z' - z'') = \langle s(z')s(z'')\rangle = \exp(-|z' - z''|/L_c). \tag{3}$$

In the following analysis we have neglected coupling between bound modes and radiation modes. This is justified[4] because the coupling between bound and radiation modes that is due to scattering from inhomogeneities is maximal for correlation lengths of the order of $\sim 1/(\beta - kn_{\text{cl}})$, where β is the propagation constant of the bound mode, k is the free-space wave number, and n_{cl} is the cladding index. Typically, for weakly guiding silica-based waveguides this length is of the order of 100 μm. This is more than two orders of magnitude longer than either the grating period or the actual roughness correlation length (which are the length scales of interest in this problem). Furthermore, this radiation coupling typically leads to an attenuation of a fraction of a decibel per centimeter. Considering the length of typical gratings, this should have a negligible effect on grating performances.

3. COUPLED-MODE FORMALISM

Using the standard approximations of coupled-mode theory, we obtain the two following coupled-mode equations[5]:

$$ia_1'(z) + \delta a_1(z) + \kappa a_2(z) = \sigma \exp[-i(2\pi/d)z]s(z)a_2(z), \quad (4a)$$

$$-ia_2'(z) + \delta a_2(z) + \kappa a_1(z) = \sigma \exp[+i(2\pi/d)z]s(z)a_1(z), \quad (4b)$$

where $a_1(z)$ and $a_2(z)$ are the slowly varying amplitudes of the forward- and backward-propagating modes defined by

$$E(x, z) = \{a_1(z)\exp[i(\pi/d)z] + a_2(z)\exp[-i(\pi/d)z]\}\psi(x), \quad (5)$$

and $\psi(x)$ is the modal profile. The terms on the right-hand side of the coupled-mode equations are the contributions from the roughness. The detuning is defined by $\delta = k\bar{n} - \pi/d$, where d is the period and $\bar{n}$ is the average modal index in the grating. The grating strength is given by $\kappa = \pi\Delta n/\lambda$, where Δn is the amplitude of the index modulation and λ is the free-space wavelength. The coupled-mode equations above and the analysis that follows also apply to nonuniform gratings represented by parameters δ and κ, which vary slowly as a function of z.

4. PERTURBATION ANALYSIS

If we assume a small standard deviation in the roughness, i.e., $\sigma \ll \kappa$, the coupled-mode equations can be solved by a perturbation series in σ,

$$a_i^{(0)}(z) + \sigma a_i^{(1)}(z) + \sigma^2 a_i^{(2)}(z) + \ldots, \quad (6)$$

by a Green-function technique. Successive terms in the perturbation series are related by the Green function as follows:

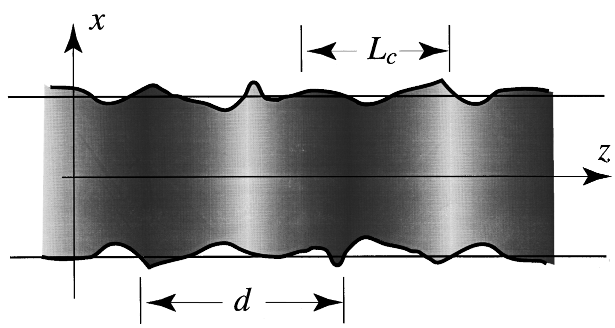

Fig. 1. Schematic representation of typical surface roughness along the vertical core–cladding interface of an etched silica-based waveguide. The correlation length of the surface roughness is L_c. The periodic shading indicates the variations in the refractive index that produce a Bragg grating with period d. Note that d and L_c are comparable in magnitude.

$$\begin{bmatrix} a_1^{(n+1)}(z) \\ a_2^{(n+1)}(z) \end{bmatrix} = \int_0^L s(z')\begin{bmatrix} G_{11}(z, z') & G_{12}(z, z') \\ G_{21}(z, z') & G_{22}(z, z') \end{bmatrix} \times \begin{bmatrix} \exp[-i(\pi/d)z']a_2^{(n)}(z') \\ \exp[+i(\pi/d)z']a_1^{(n)}(z') \end{bmatrix} dz'. \quad (7)$$

The Green function can be represented in terms of the solutions to the original unperturbed grating as given in Appendix B. With the Green function (which implicitly contains the grating boundary conditions) the reflection coefficient r can be shown to be

$$r = a_2(0) = r^{(0)} + \sigma \int_0^L s(z')F(z')dz' + \sigma^2 \int_0^L \int_0^L s(z')s(z'')J(z', z'')dz'dz'' + O(\sigma^3), \quad (8)$$

where F and J are given in Appendix B. From the reflection coefficient, the reflectance $R = |r|^2$ can be calculated. The phase of the reflected wave and information such as the group delay of reflected pulses can also be obtained from the expression for r; the details are given in Appendix C.

5. ENSEMBLE AVERAGES

We obtain the ensemble average for the reflectance R by taking the norm of Eq. (8), applying the ensemble average, and using Eq. (3):

$$\langle R \rangle = R^{(0)} + \sigma^2 \text{ Re} \int_0^L \int_0^L \Gamma(z' - z'')[F(z')F(z'')^* + 2r^{(0)*}J(z', z'')]dz'dz'' + O(\sigma^3), \quad (9)$$

where * mean complex conjugate. In order to proceed further, we explicitly separate rapidly and slowly varying parts as follows:

$$\langle R \rangle = R^{(0)} + \sigma^2 \text{ Re} \int_0^L \int_0^L \Gamma(z' - z'') \times\{A(z', z'')\exp[i(2\pi/d)(z' - z'')] + B(z', z'')\exp[i(2\pi/d)(z' + z'')]\}dz'dz'' + O(\sigma^3), \quad (10)$$

where A and B are extracted from the expressions for F and J such that they depend only on the field amplitudes and are therefore slowly varying functions of z' and z'', whereas the correlation function and the exponentials are rapidly varying functions. We have also exploited the symmetry between z' and z''. We introduce the more convenient variables $y = z' - z''$ and $z = (z' + z'')/2$, and since $\Gamma(y)$ has a sharp peak near $y = 0$, the slowly varying quantities and the inner integration can be approximated by their values at $y = 0$. Thus

$$\langle R \rangle \approx R^{(0)} + \sigma^2 \text{ Re} \int_{-L}^{L} \Gamma(y) \int_0^L A(z, z)\exp[i(2\pi/d)y] + B(z, z)\exp[i(4\pi/d)z]dz\,dy. \quad (11)$$

Since $\Gamma(y)$ also decays very rapidly with y, the range of integration can be extended to infinity and performed as a Fourier integral, giving the power spectrum $S(k)$ of the roughness. Also, the rapidly oscillating factor in the second integral allows it to be neglected. Therefore

$$\langle R \rangle \approx R^{(0)} + \sigma^2 S(2k_B) L A(\delta), \tag{12}$$

where $k_B = \pi/d$ is the Bragg wave number of the grating and

$$A(\delta) = \frac{1}{L}\int_0^L A(z, z)\mathrm{d}z$$
$$= \frac{1}{L}\,\mathrm{Re}\int_0^L \left\{ \frac{(1 - R)[P_i(z)^2 + P_2(z)^2] + 4r^{*2}a_1{}^2(z)a_2{}^2(z) - 2r^* a_1(z)a_2(z)[P_1(z) + P_2(z)]}{1 - R} \right\} \mathrm{d}z \tag{13}$$

and $P_i(z) = |a_i(z)|^2$. Note that $A(\delta)$ is a dimensionless quantity that describes the spectral variations of the perturbations that are due to roughness but can be completely calculated from the solutions $a_i(z)$ of the unperturbed grating. The expression for the time delay given in Appendix C has an analogous form.

6. DEPENDENCE ON CORRELATION LENGTH

The power spectrum $S(k)$ can be easily calculated for the exponential model

$$S(k) = \frac{2L_c}{1 + k^2 L_c{}^2}. \tag{14}$$

Thus

$$\langle R \rangle \approx R^{(0)} + \frac{2\sigma^2 L_c L}{1 + 4k_B{}^2 L_c{}^2} A(\delta). \tag{15}$$

As a function of correlation length, the perturbation is maximal when $L_c = d/2\pi$. This agrees with our expectations that the effects of roughness will be strongest when the correlation length and the grating period are comparable. The spectral dependence of the perturbations that are due to roughness are completely determined by the factor $A(\delta)$, and this dependence is examined in more detail in the following sections. As we shall see, in most cases $A(\delta)$ is of the order of unity; thus an order of magnitude estimate of the perturbation is

$$\frac{2\sigma^2 L_c L}{1 + 4k_B{}^2 L_c{}^2} \leqslant \sigma^2 L L_c, \tag{16}$$

where the expression on the right is the maximum possible value with $L_c = d/2\pi$. We can rewrite this as

$$(\sigma/\kappa)^2(\kappa L)^2(L_c/L). \tag{17}$$

Thus, although the strength of the effect increases as the square of the grating strength κL, it is also proportional to the square of the size of the perturbation $(\sigma/\kappa)^2 \approx 10^{-4}$ and to the ratio of the correlation length to the length of the grating $L_c/L \approx 10^{-5}$. Thus even for very strong gratings with $\kappa L \approx 100$ the perturbation in the reflectance is still only of the order of 10^{-5}. A similar result is obtained for the size of perturbations to the delay properties of the grating. A more rigorous calculation including the spectral variation of the perturbation is presented in the following sections.

7. RESULTS FOR A UNIFORM GRATING

The spectral variations of the perturbations in $\langle R \rangle$ are given by $A(\delta)$ and depend on the specific structure of the grating. The expression for $A(\delta)$ for a uniform grating can be evaluated exactly and is given by

$$A(\delta) = \frac{1}{4(\alpha^2\cosh^2\alpha L + \delta^2\sinh^2\alpha L)^3}\Bigg[(3\kappa^4 - 4\alpha^4)\delta^2 + \kappa^4(2\alpha^2 + 3\kappa^2)\cosh^2\alpha L + 2\kappa^4(2\alpha^2 - 3\kappa^2) \times \cosh^4\alpha L - \delta^2\kappa^2(4\alpha^2 + 9\kappa^2)\frac{\sinh 2\alpha L}{2\alpha L} + \kappa^4(9\kappa^2 - 8\alpha^2)\cosh^2\alpha L\,\frac{\sinh 2\alpha L}{2\alpha L}\Bigg], \tag{18}$$

where $\alpha = (\kappa^2 - \delta^2)^{1/2}$. The complicated expression above is shown graphically in Fig. 2. Note that $A(\delta)$ is small and negative inside the band gap ($|\delta| < \kappa$), tends to unity far from the band gap, and has a series of peaks on either side of the band gap. A careful examination reveals that these peaks occur where the reflectance of the unperturbed grating is zero ($R = 0$), corresponding to Fabry–Pérot resonances. Here we give some simplifications of the above result for four different regions: far from the Bragg resonance, in the center of the gap, at the band edge, and at the zeros of the reflectance.

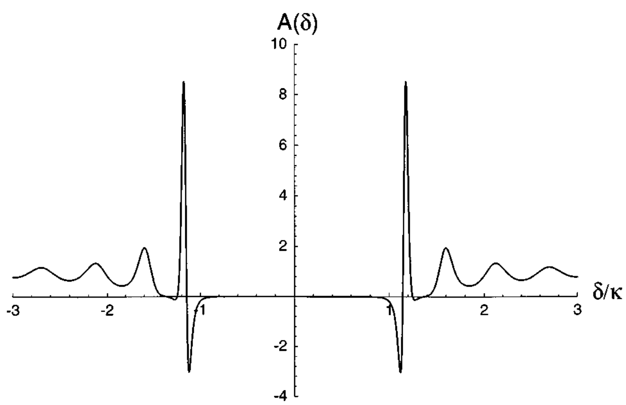

Fig. 2. Spectral dependence of roughness perturbation for a uniform grating of strength $\kappa L = 5$.

A. Large Detuning

For large detunings, i.e., when $|\delta| \gg \kappa$, the reflectance is given by

$$\langle R\rangle \approx R^{(0)} + \sigma^2 L S(2k_B), \tag{19}$$

which agrees with the result in our previous paper for backscattering in the absence of any grating[2] and shows that the roughness leads to linear attenuation with distance.

B. Zero Detuning

At zero detuning, i.e., when $\delta = 0$, the reflectance is

$$\langle R\rangle \approx R^{(0)} + \frac{\sigma^2 L}{4\cosh^4(\kappa L)}\left(4 - \cosh 2\kappa L + \frac{\sinh 2\kappa L}{2\kappa L}\right) \times S(2k_B) \sim R^{(0)} - 2\sigma^2 L \exp(-2\kappa L) S(2k_B). \tag{20}$$

The final expression gives the asymptotic behavior for strong gratings. Thus the effect of roughness inside the band gap becomes smaller as the grating gets stronger. This can be explained simply by noting that a strong grating reflects almost all the light at the front of the grating, and therefore very little of it samples the roughness.

C. Band Edge

At the band edge, i.e., when $\delta = \pm\kappa$, the reflectance is

$$\langle R\rangle \approx R^{(0)} + \frac{\sigma^2 L}{48(1+\kappa^2L^2)^3}(48 + 32\kappa^2L^2 - 13\kappa^4L^4 - 9\kappa^6L^6)S(2k_B) \sim R^{(0)} - \frac{9\sigma^2 L}{48}S(2k_B). \tag{21}$$

D. Zeros of Reflectance

For the weak Fabry–Pérot resonances that occur at $\delta = \pm(\kappa^2 + n^2\pi^2/L^2)^{1/2}$ the perturbed reflectance is given approximately by

$$\langle R\rangle \approx \sigma^2 L\left[1 - \left(\frac{\kappa L}{n\pi}\right)^2 + \frac{3}{4}\left(\frac{\kappa L}{n\pi}\right)^4\right]S(2k_B) \sim \frac{3}{4}\sigma^2 L\left(\frac{\kappa L}{n\pi}\right)^4 S(2k_B). \tag{22}$$

Although the size of the perturbation scales with the fourth power of the grating strength, the prefactor is so small, for typical parameters, that the effect on the resonances is significant only if $\kappa L > 40$. The first zero for a uniform grating of strength $\kappa L = 50$ is shown in Fig. 3, and the visibility of the fringe is reduced by only 10%. Nevertheless, this suggests that gratings with very strong Fabry–Pérot resonances and corresponding narrow spectral features may be more strongly affected by roughness.

8. RESULTS FOR A PHASE-SHIFTED GRATING

Many grating-based devices rely on the presence of very narrow spectral features such as transmission resonances. The sidelobes of a uniform grating arise from a rather weak interference effect of reflections from the ends of the grating. A much stronger interference effect occurs for a uniform grating with a single-phase discontinuity in the center of the grating. Such a structure will exhibit a sharp transmission fringe in the spectrum.

For the unperturbed phase-shifted grating the reflection at the center of the reflection band is zero. Roughness produces a reflection at this point given by

$$\langle R\rangle \approx \sigma^2 L S(2k_B)\frac{\exp(2\kappa L)}{32\kappa L}. \tag{23}$$

This expression grows exponentially with grating strength and is appreciable even for $\kappa L \approx 10$. A closeup of the narrow transmission fringe is shown in Fig. 4 for a moderate-strength grating ($\kappa L = 10$). Note the contrast of the fringe is reduced by almost 20% by the roughness.

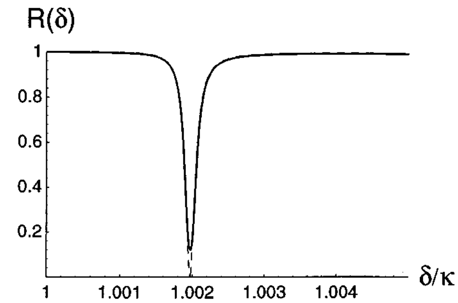

Fig. 3. Reflectance including roughness perturbation for a uniform grating of strength $\kappa L = 50$ shown in closeup near the first Fabry–Pérot resonance.

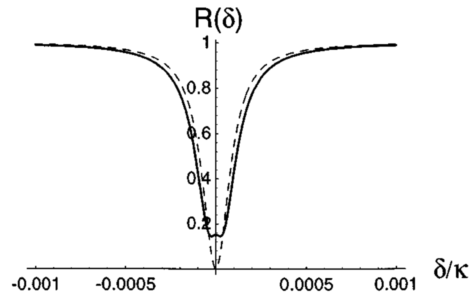

Fig. 4. Reflectance including roughness perturbation for a phase-shifted grating of strength $\kappa L = 10$ shown in a closeup near the central transmission resonance. The dashed curve is for the unperturbed grating, and the solid curve includes the effects of roughness.

9. CONCLUSION

Thus we can conclude that in most practical situations the effect of roughness on uniform gratings is negligible. For broad spectral features the size of the roughness perturbation on the reflection coefficient is of the order of

$$(\sigma/\kappa)^2(\kappa L)^2(L_c/L) \tag{24}$$

and can be estimated to be of the order of 10^{-5} for silica-based grating structures. The strongest effects occur near transmission resonances and even then very strong gratings ($\kappa L \gg 10$) are necessary to produce a noticeable change. Thus, sidewall roughness does not significantly alter the overall features of the reflectance.

On the other hand, as we have shown for a phase-shifted grating, narrow spectral features that are due to strong Fabry–Pérot resonances can be strongly affected by roughness. Thus the ensemble analysis suggests that it might be difficult to reproducibly implement structures with narrow spectral features in planar waveguide technology. This is not the case for fiber-based gratings, since sidewall roughness is insignificant.

APPENDIX A: STEP-INDEX SLAB-WAVEGUIDE COUPLING COEFFICIENT

Using a standard technique,[3] we can show that the roughness coupling coefficient is given by

$$K(z) = f(z)\left[\frac{U^2 W}{2\rho^3 \beta(1 + W)}\right], \tag{A1}$$

where the quantities in the bracket are all standard waveguide and modal parameters:

$$U = \rho(k^2 n_{\mathrm{co}}{}^2 - \beta^2)^{1/2}, \tag{A2a}$$

$$W = \rho(\beta^2 - k^2 n_{\mathrm{cl}}{}^2)^{1/2}, \tag{A2b}$$

where ρ is the waveguide half-width, β is the propagation constant of the mode, k is the free-space wave number, and n_{co} and n_{cl} are the core and the cladding indices, respectively. Thus the random variations in the coupling are proportional to the surface roughness. The standard deviation of $K(z)$ is simply related to that of $f(z)$ by

$$\sigma = \delta_f\left[\frac{U^2 W}{2\rho^3 \beta(1 + W)}\right]. \tag{A3}$$

APPENDIX B: GREEN FUNCTION AND RELATED EXPRESSIONS

The Green function for an arbitrary nonuniform grating can be expressed in terms of the grating spectra and the fields in the grating by the standard method of variation of parameters and is given by

$$\begin{aligned} G_{jk}(z, z') = \frac{-i}{1 - R^{(0)}} [& \theta(z - z') a_j^{(0)}(z) a_k^{(0)}(z')^* \\ & + \theta(z' - z) a_{3-j}^{(0)}(z)^* a_{3-k}^{(0)}(z') \\ & - r^{(0)*} a_j^{(0)}(z) a_{3-k}^{(0)}(z')], \end{aligned} \tag{B1}$$

where r and $R = |r|^2$ are the reflection coefficients and θ is the Heaviside or unit step function.

The kernel functions appearing in the perturbation integrals for r are given by

$$\begin{aligned} F(z') = & \; G_{21}(0, z')\exp[-i(2\pi/d)z'] a_2^{(0)}(z') \\ & + G_{22}(0, z')\exp[+i(2\pi/d)z'] a_1^{(0)}(z') \end{aligned} \tag{B2a}$$

$$\begin{aligned} = & -i\{a_1^{(0)}(z')^2 \exp[+i(2\pi/d)z'] \\ & + a_2^{(0)}(z')^2 \exp[-i(2\pi/d)z']\}, \end{aligned} \tag{B2b}$$

$$\begin{aligned} J(z', z'') = & \; G_{21}(0, z')G_{21}(z', z'')\exp[-i(2\pi/d) \\ & \times (z' + z'')] a_2^{(0)}(z'') + G_{21}(0, z')G_{22}(z', z'') \\ & \times \exp[-i(2\pi/d)(z' - z'')] a_1^{(0)}(z'') \\ & + G_{22}(0, z')G_{11}(z', z'') \\ & \times \exp[+i(2\pi/d)(z' - z'')] a_2^{(0)}(z'') \\ & + G_{22}(0, z')G_{12}(z', z'') \\ & \times \exp[+i(2\pi/d)(z' + z'')] a_1^{(0)}(z'') \end{aligned} \tag{B3a}$$

$$\begin{aligned} = & -i\{a_2^{(0)}(z') a_2^{(0)}(z'') G_{21}(z', z'')\exp[-i(2\pi/d) \\ & \times (z' + z'')] + a_2^{(0)}(z') a_1^{(0)}(z'') G_{22}(z', z'') \\ & \times \exp[-i(2\pi/d)(z' - z'')] + a_1^{(0)}(z') a_2^{(0)}(z'') \\ & \times G_{11}(z', z'')\exp[+i(2\pi/d)(z' - z'')] \\ & + a_1^{(0)}(z') a_1^{(0)}(z'') G_{12}(z', z'') \\ & \times \exp[+i(2\pi/d)(z' + z'')]\}. \end{aligned} \tag{B3b}$$

APPENDIX C: EXPRESSIONS FOR THE TIME DELAY

The phase of the reflected wave can be calculated from $\phi = \mathrm{Im}[\ln(r)]$. A more interesting related quantity is the time delay, which is the derivative of the phase and defined by

$$\mathscr{T} = \frac{\bar{n}}{c} \mathrm{Im}\left[\frac{\mathrm{d}}{\mathrm{d}\delta} \ln(r)\right], \tag{C1}$$

where c is the speed of light in free space.

The perturbation expression for the phase delay is

$$\begin{aligned} \langle \mathscr{T} \rangle = & \; \mathscr{T}^{(0)} + \sigma^2 \frac{\bar{n}}{c} \mathrm{Im} \frac{\mathrm{d}}{\mathrm{d}\delta} \int_0^L \int_0^L \Gamma(z' - z'') \\ & \times \left[\frac{J(z', z'')}{r^{(0)}} - \frac{F(z')F(z'')}{2r^{(0)^2}}\right] \mathrm{d}z' \mathrm{d}z'' + O(\sigma^3), \end{aligned} \tag{C2}$$

which, by a similar sequence of steps as for $\langle R \rangle$, yields

$$\langle \mathscr{T} \rangle \approx \mathscr{T}^{(0)} + \sigma^2 S(2k_B) L C(\delta), \tag{C3}$$

where

$$C(\delta) = \frac{1}{L} \frac{\bar{n}}{c} \mathrm{Im} \frac{\mathrm{d}}{\mathrm{d}\delta} \int_0^L \left\{\frac{(1 + 2R) a_1{}^2(z) a_2{}^2(z) - r a_1(z) a_2(z)[P_1(z) + P_2(z)]}{r^2(1 - R)}\right\} \mathrm{d}z. \tag{C4}$$

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