Periodic Optical Waveguides: Exact Floquet Theory and Spectral Properties

By J. A. Besley, N. N. Akhmediev, and P. D. Miller

We consider the steady propagation of a light beam in a planar waveguide whose width and depth are periodically modulated in the direction of propagation. Using methods of soliton theory, a class of periodic potentials is presented for which the complete set of Floquet solutions of the linear Schrödinger equation can be found exactly at a particular optical frequency. For potentials in this class, there are exactly two bound Floquet solutions at this frequency, and they are degenerate, having the same Floquet multiplier. We study analytically the behavior of the waveguide under small changes in the frequency and observe a breaking of the degeneracy in the Floquet multiplier at first order. We predict and observe numerically the disappearance of both bound states at second order. These results suggest applications to spectral filtering.

1. Introduction

The steady linear propagation of paraxial monochromatic beams in slowly varying, periodic, planar waveguides is described by the Floquet theory of

Address for correspondence: J. A. Besley, Australian Photonics Cooperative Research Centre, Optical Sciences Centre, Australian National University, Canberra, ACT 0200 Australia

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the nonstationary linear Schrödinger equation

$$i\phi_z + \tfrac{1}{2}\phi_{xx} + V(x,z)\phi = 0, \qquad V(x,z+L) = V(x,z), \tag{1}$$

where the given potential function $V(x,z)$ represents the spatial profile of the effective refractive index in the (x,z)-plane seen by monochromatic light of frequency ω (here, $\omega = 1$ in normalized units). For this problem, the period map $T(z)$: $\phi(x,z) \mapsto \phi(x,z+L)$ is a unitary linear operator whose eigenfunctions are beam profiles that evolve under (1) as quasiperiodic functions of z. The corresponding eigenvalues t are the Floquet multipliers written in the form $t = \exp(i\beta L)$. The numbers β, determined modulo $2\pi/L$, are then called the quasi-propagative constants analogous to the propagative constants of the stationary modal theory for potentials independent of z. If the operator T can be diagonalized on an appropriate space of functions (beam profiles), then the initial value problem for (1) can be solved, and the effects of slight shifts in the operating frequency ω can be calculated by using perturbative techniques.

For general potential functions $V(x,z)$, constructing the period map T and finding its subsequent spectral decomposition requires numerical methods. It is possible, however, to find classes of physically reasonable periodic potentials for which T admits an exact diagonalization. One such class is found within the family of so-called *separable potentials* [1]. In this article, we examine the periodic waveguides in this class that are additionally even functions of x. We find exactly two bound states within the spectrum of T. They are degenerate, having the same Floquet multiplier, and are immersed in the continuous spectrum. In Section 2 we describe in detail the class of potentials $V(x,z)$ under consideration, present their Floquet solutions, show that they are complete, and thus diagonalize the period map T. Then, in Section 3, we study the beam propagation problem for a fixed periodic potential $V(x,z)$ of the type described in Section 2 under small changes ϵ in the optical frequency ω. Numerical simulations show that there are effects on two scales. First, the degeneracy is broken, resulting in a beating of the two bound Floquet modes on a length scale $\sim \epsilon^{-1}$. Next, there is decay of both bound modes on a longer scale $\sim \epsilon^{-2}$. We seek to capture these effects through a multiple-scale analysis of the propagation problem (1). We find an accurate formula for the beat length and indicate the nature of the second-order calculation necessary to capture the modal decay. We discuss our results in Section 4.

Understanding periodic potentials of the linear Schrödinger equation (1) is of practical interest in optics principally for applications related to wavelength filtering (wavelength-division multiplexing [2]), such as dispersion compensation and signal control, and also in applications, such as pulse compression [3].

2. Separable periodic potentials

We recall some facts [1] about separable potentials for the linear Schrödinger equation (1). Let $\Phi(x, z, \lambda)$ and $H(x, z, \lambda)$ be functions of the general form

$$\begin{aligned} \Phi(x,z,\lambda) &= \left(\lambda^M + \sum_{p=0}^{M-1} \lambda^p g_p(x,z)\right) \exp(-2i(\lambda x + \lambda^2 z)), \\ H(x,z,\lambda) &= \sum_{p=0}^{M-1} \lambda^p h_p(x,z). \end{aligned} \tag{2}$$

These functions have undetermined coefficients $g_p(x, z)$ and $h_p(x, z)$. We fix them by first choosing M distinct complex numbers λ_k in the upper half plane, M proportionality constants γ_k, and then imposing the conditions

$$\begin{aligned} \Phi(x,z,\lambda_k) &= \gamma_k^* H(x,z,\lambda_k), \\ H(x,z,\lambda_k^*) &= -\gamma_k \Phi(x,z,\lambda_k^*), \end{aligned} \tag{3}$$

for $k = 1, \ldots, M$. Written out explicitly, these are $2M$ linear algebraic equations in the $2M$ unknowns $g_p(x, z)$ and $h_p(x, z)$. From the solution of these equations, define the potential function

$$V(x,z) = 4|h_{M-1}(x,z)|^2. \tag{4}$$

Then it turns out that for each complex λ, the function $\Phi(x, z, \lambda)$ is a solution of the linear Schrödinger equation

$$i\phi_z + \tfrac{1}{2}\phi_{xx} + V(x,z) = 0. \tag{5}$$

Thus by solving the system of linear algebraic equations (3) for the coefficients $g_p(x, z)$ and $h_p(x, z)$, the data set $\{\lambda_1, \ldots, \lambda_M, \gamma_1, \ldots, \gamma_M\}$ yields an explicit potential function $V(x, z)$ and a family of solutions $\Phi(x, z, \lambda)$, parametrized by λ of the linear equation (5).

The reader may observe that this algebraic procedure is essentially the construction of M-soliton solutions $\psi(x, z) = 2ih_{M-1}(x, z)$ of the nonlinear Schrödinger equation

$$i\psi_z + \tfrac{1}{2}\psi_{xx} + |\psi|^2\psi = 0. \tag{6}$$

The numbers λ_k are the soliton eigenvalues, and the numbers γ_k contain information about the soliton phases. The potential function $V(x, z)$ under consideration is just the self-consistent nonlinear potential in equation (6). This fact means that these potentials are especially easy to induce optically in a planar medium [4, 5]. The whole procedure extends easily to nonlinear Schrödinger equations for vector-valued fields with N components [1].

The family of solutions to the linear Schrödinger equation with potential $V(x,z)$ is complete, because by superposition with different values of λ, the formula for $\Phi(x,z,\lambda)$ really contains the general solution of this equation for absolutely continuous L_1 initial data $f(x)$. In fact, for any such $f(x)$ and fixed arbitrary z, we have the expansion

$$f(x) = \lim_{R\to\infty}\int_{-R}^{R} A(\lambda)\Phi(x,z,\lambda)\,d\lambda + \sum_{k=1}^{M} A_k\Phi(x,z,\lambda_k^*). \tag{7}$$

The integral is taken over real λ, where the expansion coefficients $A(\lambda)$ and A_k are obtained by using the orthogonality conditions

$$\begin{gathered}\int_{-\infty}^{\infty}\Phi(x,z,\lambda)\Phi(x,z,\mu)^*\,dx = \pi\delta(\lambda-\mu)\prod_{k=1}^{M}(\lambda-\lambda_k)(\lambda-\lambda_k^*),\\ \int_{-\infty}^{\infty}\Phi(x,z,\lambda)^*\,\Phi(x,z,\lambda_j)^*dx=0, \qquad \int_{-\infty}^{\infty}\Phi(x,z,\lambda_j^*)\Phi(x,z,\mu)^*\,dx=0,\\ \int_{-\infty}^{\infty}\Phi(x,z,\lambda_j^*)\Phi(x,z,\lambda_k)^*\,dx = \frac{\delta_{jk}}{2i}(\lambda_k^*-\lambda_k)\prod_{m\neq k}(\lambda_k^*-\lambda_m)(\lambda_k^*-\lambda_m^*).\end{gathered} \tag{8}$$

Here, λ and μ are real, and $k=1,\ldots,M$.

Now let us examine this complete set of solutions of the Schrödinger equation (1) when we take $M=2$ and select the discrete eigenvalues $\lambda_1 = ib_1$ and $\lambda_2 = ib_2$ (without loss of generality, we take $b_1 > b_2$) and proportionality constants $\gamma_1=\gamma_2=1$ to obtain a z-periodic potential $V(x,z)$. As shown previously, this potential function can be written as the square modulus of a complex function $\psi(x,z)$,

$$\psi(x,z) = \frac{4i(b_2^2-b_1^2)\left[b_1\cosh(2b_2x)\exp(2ib_1^2z)-b_2\cosh(2b_1x)\exp(2ib_2^2z)\right]}{(b_1-b_2)^2C_+(x)+(b_1+b_2)^2C_-(x)-4b_1b_2\cos D(z)}, \tag{9}$$

where $C_\pm(x)=\cosh[2(b_1\pm b_2)x]$, $D(z)=2(b_1^2-b_2^2)z$. The potential function $V(x,z)$ is periodic in z with period $L=\pi/(b_1^2-b_2^2)$. The choice $\gamma_1=\gamma_2=1$ ensures that the potential function is an even function of x for all z. The two free parameters, b_1 and b_2, control the period and shape of the potential and may be considered design parameters for a planar, periodic waveguide. For a particular choice of b_1 and b_2, the potential function is shown in Figure 1.

For this class of potential functions, all of the Floquet solutions of the problem are given by the function $\Phi(x, z, \lambda)$ evaluated at particular values of λ. Both $\Phi(x, z, -ib_1)$ and $\Phi(x, z, -ib_2)$ are (linearly independent) bound states with the same Floquet multiplier, $t = \exp(2ib_1^2 L) = \exp(2ib_2^2 L)$. For real λ, $\Phi(x, z, \lambda)$ is an unbound Floquet solution in the continuous spectrum of the period map T with multiplier $t(\lambda) = \exp(-2i\lambda^2 L)$. By completeness [1], these functions diagonalize the period map $T(z)$ for this problem.

Because the potential is an even function of x, there exist odd and even combinations of the two degenerate bound states that are themselves bound Floquet solutions. The even solution is

$$\Phi^{\text{even}}(x,z) = c_1^{\text{even}}\Phi(x,z,-ib_1) + c_2^{\text{even}}\Phi(x,z,-ib_2) = \psi(x,z), \quad (10)$$

and the odd solution is

$$\begin{aligned}\Phi^{\text{odd}}&(x,z)\\ &= c_1^{\text{odd}}\Phi(x,z,-ib_1) + c_2^{\text{odd}}\Phi(x,z,-ib_2)\\ &= \frac{4\left(b_2^2-b_1^2\right)\sqrt{b_1 b_2}\left[\sinh(2b_2 x)\exp\left(2ib_1^2 z\right)-\sinh(2b_1 x)\exp\left(2ib_2^2 z\right)\right]}{(b_1-b_2)^2 C_+(x)+(b_1+b_2)^2 C_-(x)-4b_1 b_2\cos D(z)}.\end{aligned} \quad (11)$$

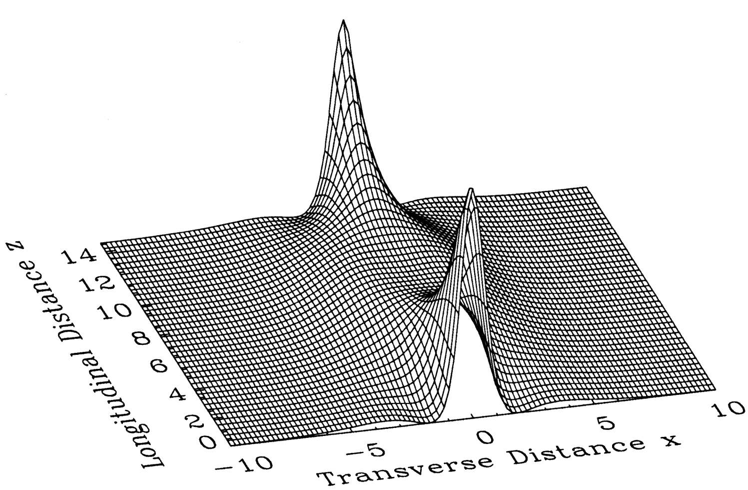


Figure 1. Periodic waveguide refractive index distribution and even mode intensity distribution. The frequency is $\omega = 1$, and the waveguide parameters are $b_1 = 0.5$ and $b_2 = 0.2$.

The coefficients are functions of the parameters b_1 and b_2. They are

$$c_1^{\text{even}} = \frac{2}{b_1 - b_2}, \qquad c_2^{\text{even}} = \frac{2}{b_2 - b_1},$$
$$c_1^{\text{odd}} = \sqrt{\frac{b_2}{b_1}}\,\frac{2}{b_1 - b_2}, \qquad c_2^{\text{odd}} = \sqrt{\frac{b_1}{b_2}}\,\frac{2}{b_2 - b_1}. \tag{12}$$

The intensity profiles of these two modes are shown in Figures 1 and 2. Exploiting the evenness of the potential $V(x,z)$, the expansion formula [1] becomes

$$f(x) = \lim_{R\to\infty}\int_{-R}^{R} A(\lambda)\Phi(x,z,\lambda)\,d\lambda + A_{\text{even}}\Phi^{\text{even}}(x,z) + A_{\text{odd}}\Phi^{\text{odd}}(x,z), \tag{13}$$

where the expansion coefficients are determined in terms of $f(x)$ using the orthogonality conditions

$$\int_{-\infty}^{\infty}\Phi(x,z,\lambda)\Phi(x,z,\mu)^*\,dx = \pi\delta(\lambda-\mu)(\lambda^2+b_1^2)(\lambda^2+b_2^2),$$
$$\int_{-\infty}^{\infty}\Phi(x,z,\lambda)\Phi^{\text{even,odd}}(x,z)^*\,dx = 0, \tag{14}$$
$$\int_{-\infty}^{\infty}|\Phi^{\text{even,odd}}(x,z)|^2\,dx = 4(b_1+b_2),$$

and, of course, Φ^{even} and Φ^{odd} are mutually orthogonal.

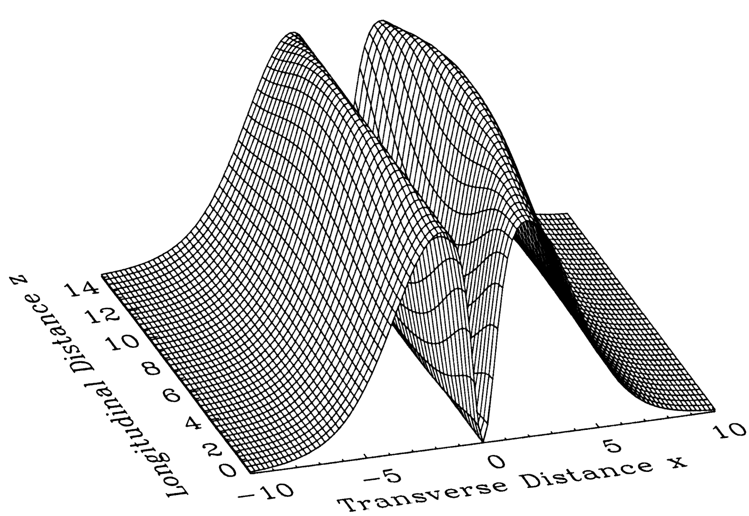


Figure 2. Odd mode intensity distributions. The frequency is $\omega = 1$, and the waveguide parameters are $b_1 = 0.5$ and $b_2 = 0.2$.

3. Spectral analysis

There is a structural instability of the bound Floquet modes that suggests the application of this family of periodic waveguides for spectral filtering. This is because a fixed refractive index profile gives rise to a different linear Schrödinger equation for each operating frequency ω. We have normalized the frequency parameter to the value 1 in the Equation (1), but upon considering a general frequency ω expressed in these units one obtains

$$i\omega\phi_z + \tfrac{1}{2}\phi_{xx} + \omega^2 V(x,z)\phi = 0. \tag{15}$$

For this periodic Schrödinger equation, the period map T depends parametrically on ω. We know that for $\omega = 1$, there is a bound state eigenvalue t of multiplicity two embedded in the continuous spectrum of T. This situation is unstable to small changes in the optical frequency ω. As illustrated in Figure 3, there are two dominant effects that appear upon introducing a small shift ϵ in the frequency, so that $\omega = 1 + \epsilon$. First, the Floquet multipliers have split, introducing a slow beating of an asymmetric initial beam profile on a length scale $\sim \epsilon^{-1}$. Next, there is decay of both bound Floquet modes due to coupling to radiation on the length scale $\sim \epsilon^{-2}$. The figure is meant only

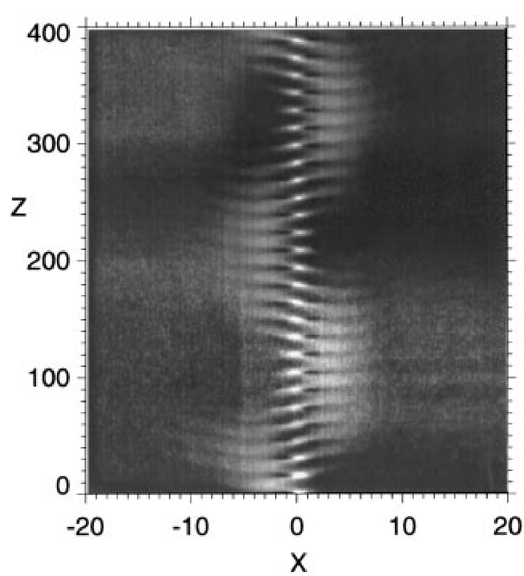


Figure 3. The beam propagation problem at frequency $\omega = 1 + \epsilon$ with $\epsilon = 0.1$ for a periodic waveguide having parameters $b_1 = 0.5$ and $b_2 = 0.2$. The initial excitation is an asymmetrical function of x. There appear to be three length scales in the picture. First, there is the scale of the waveguide period. Then there is the longer scale of the beat length between the even and odd modes. Finally, there is the yet longer scale of the decay of both modes by coupling to radiation.

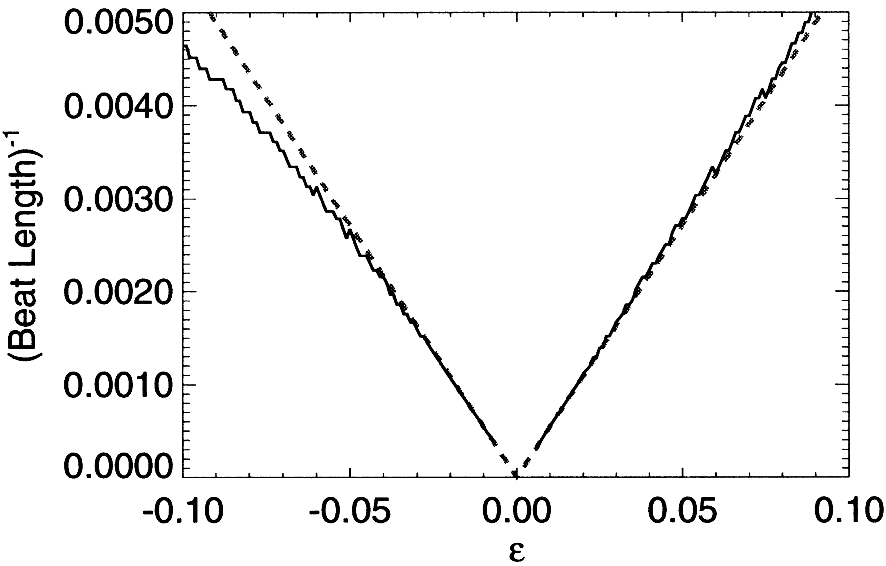


Figure 4. The inverse beat length of an asymmetric initial condition as a function of ϵ for numerical beam propagation at frequency $\omega = 1 + \epsilon$ for a periodic waveguide with parameters $b_1 = 0.5$ and $b_2 = 0.2$. Dashed line represents a straight line to fit the data.

to illustrate these effects. More convincing evidence is given in Figures 4, 5, and 6. Figure 4 compiles the results of several numerical simulations of the type shown in Figure 3, showing that the beat length for small ϵ varies as $|\epsilon|^{-1}$. Figures 5 and 6 show that the bound beam power P (which is accurately represented by the integral of the field intensity $|\phi|^2$ over the finite numerical domain $-40 < x < 40$ with radiation-field damping at the boundaries) is slowly decaying exponentially, with decay constant that varies

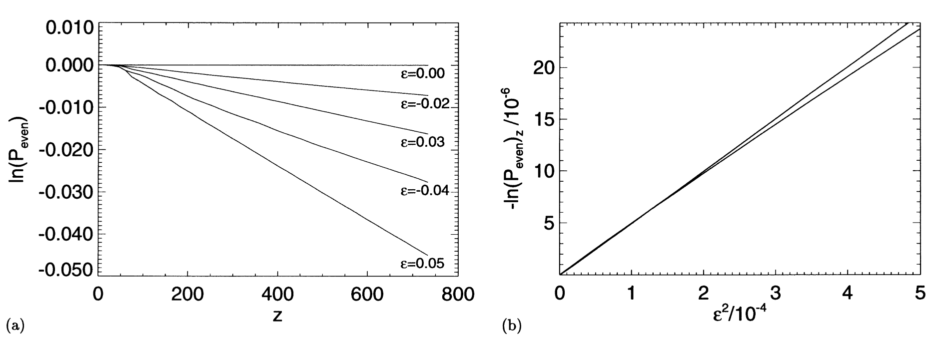
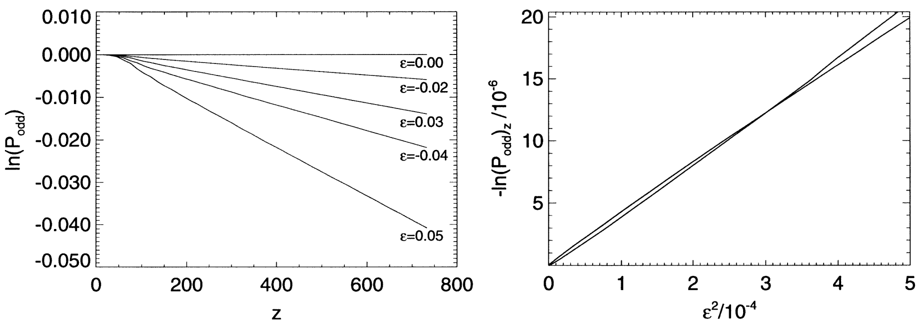


Figure 5. (**a**) The logarithm of the bound beam power P as a function of propagation distance for numerical beam propagation at several indicated values of frequency $\omega = 1 + \epsilon$ for an even initial excitation. (**b**) The numerically observed exponential decay rate (i.e., slopes of the graphs in part (**a**)) of the even mode as a function of ϵ^2 for beam propagation at frequency $\omega = 1 + \epsilon$ in a periodic waveguide with parameters $b_1 = 0.5$ and $b_2 = 0.2$.



(a) (b)

Figure 6. (**a**) The logarithm of the bound beam power P as a function of propagation distance for numerical beam propagation at several indicated values of frequency $\omega = 1 + \epsilon$ for an odd initial excitation. (**b**) The numerically observed exponential decay rate (i.e., slopes of the graphs in part (**a**)) of the odd mode as a function of ϵ^2 for beam propagation at frequency $\omega = 1 + \epsilon$ in a periodic waveguide with parameters $b_1 = 0.5$ and $b_2 = 0.2$.

as ϵ^2. Thus, the decay via loss to radiation occurs on a much longer length scale than the modal beating effect.

These numerical results suggest that the periodic waveguides described previously have no bound Floquet modes at all for most frequencies. Thus any power introduced into the waveguide will generally be lost to radiation over a long distance unless its frequency is equal to 1 in normalized units. We now investigate these multiscale phenomena, related to the structural instability of the discrete Floquet eigenvalues with respect to small changes in the operating frequency, by finding and studying the coupled mode equations.

Set $\omega = 1 + \epsilon$, and consider $\epsilon \ll 1$. We use the method of multiple scales to study the large z behavior of the bound modes. Introduce the slow scale $Z = \epsilon z$, so that the equation becomes

$$i(1+\epsilon)(\phi_z + \epsilon\phi_Z) + \tfrac{1}{2}\phi_{xx} + (1+2\epsilon+\epsilon^2)V(x,z)\phi = 0. \tag{16}$$

Assume the power series expansion,

$$\phi = \phi^{(0)} + \epsilon\phi^{(1)} + \dots, \tag{17}$$

and substitute into the Schrödinger equation, collecting like powers of ϵ. The leading order equation is one that we can solve exactly. The general solution is obtained by using the expansion formula (13),

$$\phi^{(0)} = \lim_{R\to\infty}\int_{-R}^{R} A(Z,\lambda)\Phi(x,z,\lambda)\,d\lambda + A_{\text{even}}(Z)\Phi^{\text{even}}(x,z) + A_{\text{odd}}(Z)\Phi^{\text{odd}}(x,z). \tag{18}$$

The equation at order ϵ is

$$i\phi_z^{(1)} + \tfrac{1}{2}\phi_{xx}^{(1)} + V(x,z)\,\phi^{(1)} = -i\phi_z^{(0)} - i\phi_Z^{(0)} - 2V(x,z)\,\phi^{(0)}. \tag{19}$$

By completeness of the Floquet modes for each z, we can express the solution in the form

$$\phi^{(1)} = \lim_{R\to\infty}\int_{-R}^{R} B(z,\lambda)\,\Phi(x,z,\lambda)\,d\lambda + B_{\text{even}}(z)\,\Phi^{\text{even}}(x,z) + B_{\text{odd}}(z)\,\Phi^{\text{odd}}(x,z). \tag{20}$$

The coefficients in this expansion depend on z and may additionally depend upon slow scale Z, but we will not need this latter scale for our analysis. Substituting into (19) and using the fact that for each λ the function $\Phi(x,z,\lambda)$ solves the unperturbed problem gives

$$\begin{aligned}
&\lim_{R\to\infty}\int_{-R}^{R} iB_z(z,\lambda)\,\Phi(x,z,\lambda)\,d\lambda + iB'_{\text{even}}(z)\,\Phi^{\text{even}}(x,z) \\
&\quad + iB'_{\text{odd}}(z)\,\Phi^{\text{odd}}(x,z) \\
&\quad = \lim_{R\to\infty}\int_{-R}^{R}\big[A(Z,\lambda)\big(-i\Phi_z(x,z,\lambda) - 2V(x,z)\,\Phi(x,z,\lambda)\big) \\
&\qquad\qquad - iA_Z(Z,\lambda)\,\Phi(x,z,\lambda)\big]\,d\lambda \\
&\qquad + A_{\text{even}}(Z)\big(-i\Phi_z^{\text{even}}(x,z) - 2V(x,z)\,\Phi^{\text{even}}(x,z)\big) \\
&\qquad - iA'_{\text{even}}(Z)\,\Phi^{\text{even}}(x,z) \\
&\qquad + A_{\text{odd}}(Z)\big(-i\Phi_z^{\text{odd}}(x,z) - 2V(x,z)\,\Phi^{\text{odd}}(x,z)\big) \\
&\qquad - iA'_{\text{odd}}(Z)\,\Phi^{\text{odd}}(x,z).
\end{aligned} \tag{21}$$

The individual modes can be separated by orthogonality. In particular, multiplying by $\Phi^{\text{even,odd}}(x,z)^*$ and integrating in x using the evenness of $V(x,z)$ gives

$$\begin{aligned}
&4i(b_1+b_2)\,B'_{\text{even,odd}}(z) \\
&\quad = \lim_{R\to\infty}\int_{-R}^{R} A(Z,\lambda)\int_{-\infty}^{\infty}\big(-i\Phi_z(x,z,\lambda) - 2V(x,z)\,\Phi(x,z,\lambda)\big) \\
&\qquad \times \Phi^{\text{even,odd}}(x,z)^*\,dx\,d\lambda \\
&\qquad + A_{\text{even,odd}}(Z)\int_{-\infty}^{\infty}\big(-i\Phi_z^{\text{even,odd}}(x,z) - 2V(x,z)\,\Phi^{\text{even,odd}}(x,z)\big) \\
&\qquad \times \Phi^{\text{even,odd}}(x,z)^*\,dx \\
&\qquad - 4i(b_1+b_2)\,A'_{\text{even,odd}}(Z).
\end{aligned} \tag{22}$$

Similar equations can be found for the mode amplitudes of the continuous spectrum $B(z,\lambda)$. The right-hand side of each of these equations is quasiperiodic in z for fixed Z. Linear growth of the mode amplitudes in z is avoided by choosing the dependence of $A_{\text{even,odd}}(Z)$ on the slow variable Z so that the average value (in z) of the right-hand side vanishes. Because of generic mismatch of the Floquet multipliers, the average value of the x-integral appearing within the λ-integral vanishes for almost every λ. The discrete spectrum terms, however, are strictly periodic with period $L = \pi/(b_1^2 - b_2^2)$, so we obtain as a solvability condition the equations

$$iA'_{\text{even,odd}}(Z) + \delta\beta_{\text{even,odd}} A_{\text{even,odd}}(Z) = 0, \tag{23}$$

where the shifts in quasi-propagation constant $\delta\beta_{\text{even,odd}}$ depend on b_1 and b_2 and are given by

$$\delta\beta_{\text{even,odd}} - \frac{b_1 - b_2}{4\pi} \int_0^L \int_{-\infty}^{\infty} \left(-i\Phi_z^{\text{even,odd}}(x,z) - 2V(x,z)\Phi^{\text{even,odd}}(x,z)\right)$$
$$\times \Phi^{\text{even,odd}}(x,z)^* \, dx dz. \tag{24}$$

Equation (23), along with a similar set of equations for $A(Z,\lambda)$ obtained as a solvability condition for avoiding secular growth of the amplitude $B(z,\lambda)$, makes up the coupled-mode equations for the perturbed Schrödinger equation.

Thus, at length scales of order $\sim \epsilon^{-1}$, there is no coupling of the bound states to any other modes. However, the degeneracy in the Floquet multipliers is broken at order ϵ, leading to a beat length for an asymmetric initial profile that is approximately

$$L_{\text{beat}} = \left| \frac{2\pi\epsilon^{-1}}{\delta\beta_{\text{even}} - \delta\beta_{\text{odd}}} \right|. \tag{25}$$

These findings concur with our previous numerical observations displayed in Figure 4. In fact, for the specific parametric values $b_1 = 0.5$ and $b_2 = 0.2$, formula (25) predicts an inverse beat length of $1/L_{\text{beat}} = 0.0543|\epsilon|$, which is in excellent agreement with the straight line fit to the numerical results in Figure 4. In fact, a scaling argument shows that the *normalized* beat length, $\Lambda_{\text{beat}} = L_{\text{beat}}/L$, depends only on the ratio of parametric values $b = b_2/b_1$ [6]. Therefore the normalized beat length is a one parameter function ($\Lambda_{\text{beat}}(b)$). A plot of the function $\Lambda_{\text{beat}}(b)$, which can be used to describe the beating between the odd and even Floquet modes of periodic waveguides for any choice of the parameters b_1 and b_2, is given in [6].

The numerical evidence of Figures 5 and 6 suggests that to capture the decay of the bound modes as they couple to continuous spectrum Floquet modes of the unperturbed problem, we need to go to higher order, introducing another slow scale $\tilde{Z} = \epsilon^2 z$ and seeking a solvability condition determining the leading order mode amplitudes as functions of $\tilde{Z}$. This calculation will be presented in a future publication [7]. The scaling argument described previously can be applied in this case also, so that the decay of the bound modes per waveguide period depends only on the ratio $b = b_2 / b_1$.

4. Conclusions

In the previous paragraphs, we have seen that soliton theory leads to a class of z-periodic Schrödinger potentials $V(x, z)$ that have two degenerate bound Floquet modes. Because the potential is equivalent to the self-consistent nonlinear potential of a two-soliton "breather" solution ψ of the nonlinear Schrödinger equation, the existence of the even bound state, $\Phi^{\text{even}} = \psi$, can be anticipated. The existence of the odd bound state and its degeneracy with the even bound state, however, is a new result.

The degeneracy of the modes, an effect that is strictly not possible in a waveguide with only one transverse dimension whose potential function is independent of z, means that any superposition of the odd and even bound modes will be recovered at each period of the waveguide. This fact immediately suggests an optical application. A linear periodic planar optical waveguide of the type described in this article, operating at a frequency $\omega = 1$ in normalized units, can be used as a kind of telescope to accurately image a one-parameter family of beam profiles over long distances z.

Having any bound Floquet modes at all in a periodic potential $V(x, z)$ is somewhat unusual, and one consequence of this is that under perturbation of the frequency parameter ω, the pair of bound modes present for $\omega = 1$ disappear altogether. If the perturbation is small, the loss of the bound modes is a second-order effect, dominated by a first-order, unimodular splitting of the degenerate Floquet multipliers of the modes. Thus, slight changes in frequency introduce slow modal beating, a fact that suggests that such a waveguide, when illuminated with an asymmetrical beam, can be used as a frequency detector. The operating frequency is determined by the location of the peak intensity of the beam after a fixed number of waveguide periods. Larger perturbations of the operating frequency will lead to the immediate destruction of the bound modes. Thus the waveguide can also be viewed as a narrow bandwidth spectral filter, taking a broad, bandwidth-bound, input beam and scattering all frequency components not close to $\omega = 1$. Some of the optical applications of these periodic structures are described in more detail in [5].

We suspect that waveguides that have properties even more remarkable than the periodic ones described here can be found in the family of separable potentials connected with soliton theory. We plan to investigate these waveguides in the future.

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