Self-similar evolution of self-written waveguides

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Numerical simulations show that channel waveguides can be self-written in photosensitive materials. As the waveguide evolves, its shape remains approximately constant, even though its depth and width change. We find an exact solution that describes this evolution, which we show to be self-similar. A wide variety of single-peaked beams form waveguides that converge to this solution.

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It has been shown experimentally that waveguides can be self-written in photopolymers[1] and UV-cured epoxy.[2] Numerical simulations[3] indicate that fairly uniform channel waveguides can also be self-written in photosensitive glasses. We have shown theoretically that the refractive-index changes that occur in photosensitive glass are large enough to form self-written channel waveguides as long as the input beam is not too narrow.[3] The only existing analytical description of this process uses series expansions to describe features that are precursors to waveguide formation.[4] However, the series cannot describe the subsequent formation of the channel waveguide. Here we use similarity techniques[5] to study this stage of the evolution, which leads to what is to our knowledge the first exact solution to the differential equations describing self-writing. We also show numerically that, for a wide range of input beams, the self-written channel evolves toward this solution, which suggests that it is stable.

Similarity techniques have been used to study Hill gratings, which are formed by internal writing in a photosensitive fiber,[6] and to describe light propagation in an inverted two-level medium.[7] Menyuk *et al.*[8] also used similarity methods to study stimulated Raman scattering, and they suggest that similarity techniques are generally useful for systems with memory. Photosensitive glasses and photopolymers do exhibit such a memory. We consider photosensitive materials that experience permanent increases in refractive index as a result of exposure at specific wavelengths. Self-written waveguides can be written in such materials as follows.[3] A single-peaked symmetric beam incident upon the material initially diffracts. The refractive index increases, increasing most in regions of high intensity, and so the index change is greatest on the propagation axis. Hence the beam begins to be guided by the refractive-index change that it has created. Over time the refractive-index structure becomes more nearly uniform, and this channel waveguide continues to evolve, becoming deeper and narrower.[3]

We consider self-writing in a planar geometry. Two equations describe self-writing; the paraxial wave equation describes the light propagation,[4,9]

$$i\,\frac{\partial E}{\partial Z} + \frac{1}{2}\,\frac{\partial^2 E}{\partial Y^2} + NE = 0\,, \tag{1}$$

and the photosensitive evolution of the refractive index in a p-photon process is described by[3,4,6]

$$\frac{\partial N}{\partial T} = (EE^*)^p\,. \tag{2}$$

Here T, Y, and Z are the normalized time, transverse coordinate, and propagation distance and $N(Y,Z,T)$ and $E(Y,Z,T)$ are the normalized refractive-index change and amplitude of the electric field envelope. These quantities are related to the corresponding physical quantities t, y, z, Δn, $\mathcal{E}$ by[4] $T = a^2 {k_0}^2 n_0 A(\mathcal{E}_0 {\mathcal{E}_0}^*)^p t$, $Y = y/a$, $Z = z/(k_0 n_0 a^2)$, $N = a^2 {k_0}^2 n_0 \Delta n$, and $E = \mathcal{E}/\mathcal{E}_0$, where a is the beam width, k_0 is the free-space wave number, n_0 is the initial index, and $\mathcal{E}_0$ is the maximum electric field amplitude. A is a measure of the degree of photosensitivity of the material. In our numerical simulations below of Eqs. (1) and (2), $E(Y,0,T)$ is Gaussian for all T, and the index is initially spatially uniform.

Because the structure evolves into a fairly uniform channel, we look for solutions where N is independent

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of Z. We also take the input beam $M(Y,T)$ to be a mode of this channel, and hence the field is of the form

$$E(Y,Z,T) = M(Y,T)\exp[i\beta(T)Z], \tag{3}$$

where β is the propagation constant and M can be taken real. As the input beam is a mode, it changes as the waveguide evolves. We also choose to maintain constant input power:

$$\mathcal{P} = \int_{-\infty}^{\infty} M(Y,T)^2 \mathrm{d}Y. \tag{4}$$

Numerical results show that as the waveguide evolves, its shape appears to remain approximately constant, even though its depth and width change.[3] This result motivates the use of self-similarity techniques.[5] Self-similar solutions depend only on certain combinations of the original variables, thus reducing the degrees of freedom of the system. We have found that the combination

$$\tilde{Y} = Y\phi(T) \tag{5}$$

is a universal parameter in this problem that can be interpreted as a time-dependent scaling of the transverse coordinate and that allows us to rewrite the system as ordinary differential equations in $\tilde{Y}$ alone. We write M and N as products of functions of T and $\tilde{Y}$. Substituting these products and Eq. (3) into Eqs. (1) and (2), and requiring consistency in Eqs. (1), (2), and (4), we find that

$$M(Y,T) = \phi(T)^{1/2}\tilde{M}(\tilde{Y}), \tag{6a}$$

$$N(Y,T) = \phi(T)^2\tilde{N}(\tilde{Y}), \tag{6b}$$

$$\beta(T) = \phi(T)^2, \tag{6c}$$

where the scaling $\phi(T)$ is given by

$$\phi'(T) = \left(\frac{\mathcal{P}}{\tilde{\mathcal{P}}}\right)^p \phi(T)^{p-1} \tag{7}$$

and $\tilde{\mathcal{P}} = \int_{-\infty}^{\infty} \tilde{M}(\tilde{Y})^2 \mathrm{d}\tilde{Y}$ is the input power in this reduced system. Here $\tilde{M}$ and $\tilde{N}$ are shapes of the modal profile and the refractive index, which satisfy

$$\frac{1}{2}\tilde{M}''(\tilde{Y}) + [\tilde{N}(\tilde{Y}) - 1]\tilde{M}(\tilde{Y}) = 0, \tag{8a}$$

$$2\tilde{N}(\tilde{Y}) + \tilde{Y}\tilde{N}'(\tilde{Y}) = \tilde{M}(\tilde{Y})^{2p}. \tag{8b}$$

One can also find the reduced system by looking for scaling symmetries of Eqs. (1) and (2) subject to Eq. (4).[5]

We determined the shapes of the mode and the refractive index by solving Eqs. (8) numerically, and they are shown in Fig. 1 for $p = 1$. For $p = 1$ we find that $\tilde{\mathcal{P}} = 2.89$; and for $p = 2$, $\tilde{\mathcal{P}} = 2.94$. The solutions in Fig. 1 are the only single-peaked solutions that decay as $\tilde{Y} \to \pm\infty$. For both $p = 1$ and $p = 2$ we found that the tails of the self-similar index profile decay as $\tilde{Y}^{-2}$ and that the tails of the mode decay exponentially. Combined with scalings (6), the solution in Fig. 1 is to our knowledge the first exact solution to the self-writing problem.

Recall that when we derive the solution in Fig. 1, the input beam is the mode of the evolving waveguide, and so it is time varying. We now investigate whether the self-similar evolution can be related to the evolution of a waveguide produced by a time-dependent input beam, which we take to be Gaussian.[3] Numerical simulations indicate that the shape of the index profile in the uniform waveguide region agrees well with Fig. 1. To test further the relationship between our solution and the simulations, we now investigate the scalings in Eqs. (6).

For $p = 1$, Eq. (7) gives $\tilde{Y} \propto YT$, so Eq. (6b) predicts that the width w of the channel scales as T^{-1}. The numerical simulation results shown in Fig. 2 confirm that $w^{-1} \propto T$ at different positions Z. The superior linearity at large Z is expected, as simulations[3] indicate that the waveguide is more nearly uniform at larger Z.

For $p = 2$, Eq. (6b) predicts that $\ln w \propto T$. This is confirmed in Fig. 3, which shows $\ln w$ versus T at different Z positions. Numerical results for the peak refractive index also confirm scalings (6b). Indeed, our numerical simulations indicate that the self-writing of waveguide according to Eqs. (1) and (2) appears to be well described by the self-similar solutions. For brevity, the remainder of the results presented here are for $p = 1$.

We now investigate scaling prediction (6c) for the propagation constant. Simulations indicate that as a self-written waveguide evolves, it becomes multimoded. The modes of this waveguide beat, producing intensity maxima that move rapidly as the waveguide

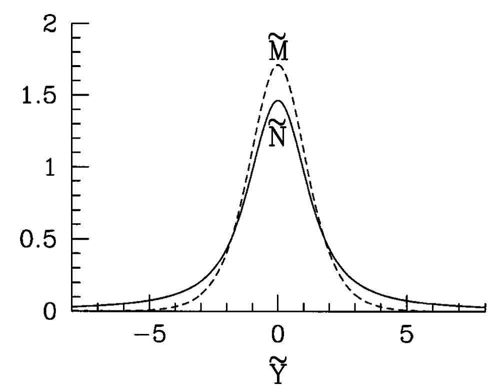

Fig. 1. Similarity solutions of the self-writing problem ($p = 1$). $\tilde{M}$ is the mode shape and $\tilde{N}$ is the refractive-index shape.

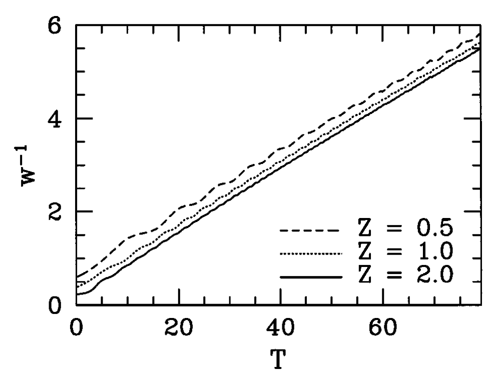

Fig. 2. Inverse width of the refractive index versus T at various values of Z ($p = 1$; simulation results).

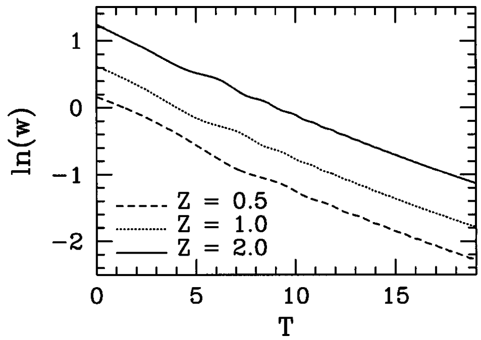

Fig. 3. Width of the refractive index versus T on a log-linear plot at various values of Z ($p = 2$; simulation results).

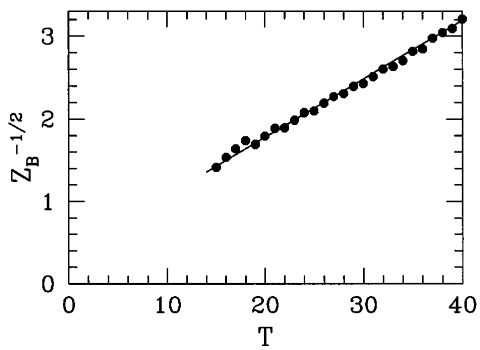

Fig. 4. $Z_B^{1/2}$ versus T ($p = 1$; simulation results), which gives a straight line, in agreement with the similarity prediction. The solid line is a straight-line fit to the data.

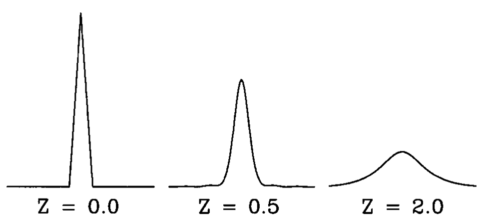

Fig. 5. Shape of the refractive-index profile for a triangular input beam at different Z ($p = 1$; simulation results).

evolves.[3] This movement allows the refractive index to remain approximately uniform in Z, even though the intensity distribution is not uniform in Z at any time.

Scalings (6) remain approximately valid when more than one mode is present and are the same for each mode. Hence the beat length Z_B scales as T^{-2} for $p = 1$. Figure 4 shows $Z_B^{-1/2}$ versus T as given by the numerical simulation. This gives an approximately straight line, as predicted. No data are shown for $T \lesssim 15$ because the waveguide then does not support more than one symmetric mode.

We thus found not only that a self-similar solution exists but also that it describes simulations of the full self-writing process well. This is remarkable: The input beam for the numerical simulation is a time-independent Gaussian, whereas the self-similar solution has a time-varying input. Note that the $\tilde{Y}^{-2}$ decay of the index profile implies that this system differs qualitatively from the seemingly related problem of spatial solitons.[10]

The index profile is initially Gaussian, and its evolution into the predicted shape suggests that the self-similar solution is stable. We also investigated a triangularly shaped input intensity. Figure 5 shows the refractive index at different positions at a particular time. Although the profile is triangular at the input face, it evolves into the self-similar solution at larger Z. For $Z \gtrsim 2.0$ the channel is approximately uniform, and hence the shape of the profile is nearly constant. This further confirms the stability of the self-similar solution and its relevance to a wide variety of self-writing processes.

It should be possible to observe this self-similar behavior experimentally in photosensitive planar waveguides. In self-writing experiments, the intensity at the output edge of the material is monitored. As a self-written waveguide evolves, similarity scaling (6a) predicts how the width and the peak values of the intensity vary with time.

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