Zero-crosstalk junctions made from dark solitons

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The bound state solutions of the linear Schrödinger equation describing steady propagation of a signal in a planar waveguide are explicitly obtained for the case of a refractive index profile created by the interaction of N arbitrary dark solitons in a defocusing Kerr medium. Analysis of these solutions shows that linear signals confined to the arms of the waveguide prior to the collision do not interfere with each other at all as they pass through the interaction region of the dark solitons. Each signal emerges from the collision with exactly the same power and angle as it had at the input, thus behaving as though all solitons other than the one in which it was initially confined were altogether absent.

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I. INTRODUCTION

It is well known that a steady intense light beam in a nonlinear medium induces a change in the background refractive index, and it has been suggested that this adjusted index profile might be used as a waveguide for less intense light beams. These beams would be guided by the nonlinearly induced index profile without significantly altering the waveguide. In the laboratory, weak beams have already been successfully guided by strong beams forming both bright [1] and dark [2,3] spatial solitons of Kerr-type nonlinear media.

The case of a Kerr-type medium in a planar geometry is particularly interesting in the context of using a strong beam to guide a weaker signal because the strong beam is modeled by an integrable cubic nonlinear Schrödinger equation, whose solutions are well understood and include radiation-free collisions of bright solitons in the self-focusing case and dark solitons in the self-defocusing case. In both of these cases, the nonlinearly induced refractive index profile of an N-soliton collision separates well before and after the interaction region of the solitons into N isolated waveguides (arms). One also might expect that the powerful mathematical tools associated with the complete integrability of the model for the strong beam could be brought to bear on the associated problem of the copropagation of weak signal beams. This is indeed the case.

There are several strategies for introducing the weak signal beam to be guided by multisoliton waveguides of the type described above. In one scenario, the weak beam coexists in the medium with the strong beam. This method is attractive because it makes possible dynamically controlled all-optical switching; the waveguide is changed as desired simply by adjusting the strong beams in the uniform bulk Kerr medium. For the signal beam to be distinguished from the pump beam, the two beams must either differ in frequency or have orthogonal polarizations. In another scenario, the index profile induced by the strong beam is fixed in the medium so that it remains in the absence of the strong beam. This could be done either by using some material with "memory" such as a photorefractive crystal, or by directly fabricating a permanent linear device by programming a computer with the appropriate exact analytical solution of the nonlinear Schrödinger equation in order to control the intensity of an ion implantation process. With this method there is no problem with using a signal beam of any frequency or polarization, but the device is then permanent.

Prior investigations [4] have shown that there are significant advantages to choosing the frequency of the signal beam to match that of the pump. If the signal and pump share the same frequency, then signal beams initially confined to the arms of the soliton waveguide remain confined after the junction, with no loss to radiation. Operating a nonlinearly induced waveguide in this strictly monochromatic regime requires that the signal and pump beams be orthogonally polarized if the two are to coexist in the medium and be distinguished from each other.

The mathematical model of the pump and signal beams is the same whether one considers orthogonally polarized beams coexisting at the same frequency in an appropriate Kerr medium, or whether one takes the pump beam to be altogether absent (although its induced index is present). For concreteness, we take both beams to be present, in which case the electric field in the slab of Kerr medium is

$$\mathbf{E}(x,z,t)=[\mathbf{e}_p\psi(X,Z)+\eta\mathbf{e}_s\phi(X,Z)]\exp[i(\beta z-\omega t)]+\text{c.c.},\tag{1}$$

where $X=\varepsilon x$, $Z=\varepsilon z$, and ε is the small ratio between the wavelength of the carrier wave and the characteristic length scale of the modulation. The orthogonal unit vectors indicating the linear polarizations of the pump and signal are $\mathbf{e}_p$ and $\mathbf{e}_s$, and the corresponding complex envelopes are $\psi(X,Z)$ and $\phi(X,Z)$. The small parameter η is the amplitude ratio of the signal to the pump. In dimensionless units, the equations for the envelopes take the form

$$i\partial_Z\psi+\tfrac{1}{2}\partial_X^2\psi-\sigma|\psi|^2\psi=0,\tag{2}$$

$$i\partial_Z\phi+\tfrac{1}{2}\partial_X^2\phi-\sigma|\psi|^2\phi=0,\tag{3}$$

where $\sigma=-1$ indicates a self-focusing medium, in which nonlinearity counteracts diffraction, and $\sigma=+1$ indicates a self-defocusing medium, in which nonlinearity enhances diffraction. The effect of the weak signal on the pump is assumed to be negligible, and the frequency ω and material

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nonlinearity are assumed to be selected so that the four-wave mixing contribution to (3), proportional to $\psi^2\bar{\phi}$, may be omitted (as would be the case if the Kerr effect is due to an electrostrictive nonlinearity). Thus, a solution $\psi(X,Z)$ of (2) induces a waveguide with an effective shift, proportional to $-\sigma|\psi(X,Z)|^2$, in the refractive index profile in the material that is seen by the signal beam. Given a particular solution ψ of (2), the mathematical task is the analysis of the solutions to the associated linear problem (3).

As pointed out above, the simplest and potentially most useful exact solutions of the nonlinear problem (2) correspond to the interactions of N bright ($\sigma=-1$) or dark ($\sigma=+1$) solitons. In both cases the arms of the induced waveguide are individually single-moded away from the junction for signals of the pump frequency. The solutions of the linear equation (3) have been studied by numerical [5,6] and approximate [7] methods in both focusing and defocusing cases for the solutions of (2) corresponding to collisions of two solitons of equal amplitudes. These results have been recently generalized somewhat in [8], where the bound states of (3) for index profiles created by completely arbitrary collisions of any number of bright solitons ($\sigma=-1$) were obtained analytically and analyzed to yield simple exact formulas for the transfer matrices that describe how a signal beam confined to a single arm of the soliton waveguide before the interaction is divided among the arms of the waveguide at the output. This analysis revealed the remarkable fact that the transfer characteristics of the N bright soliton waveguide are independent of all details of the soliton collision, depending only on the amplitudes of the solitons and their relative angles in the medium.

If the index profile is created by a collision of two dark solitons of equal amplitudes, the studies [5,7] of (3) suggest an even more remarkable phenomenon: a signal beam input into one arm of the waveguide will pass through the collision region without any scattering and with all power in the unique output arm whose angle matches that of the input arm. One may certainly ask whether this compelling result does in fact hold for waveguides made from any number of dark solitons of any amplitudes.

The remainder of this paper is dedicated to definitively answering this question by analytically considering soliton waveguides made in the defocusing case ($\sigma=+1$), where we assume that the strong field ψ consists of N arbitrary dark solitons propagating on a plane wave background (see Fig. 1 for an example of a refractive index profile nonlinearly induced by a collision of dark solitons). Of course, there is one obvious exact solution of the linear equation (3), namely, $\phi=\psi$. However, this exact solution is not as relevant as one might like, since it is not a bound state (finite power beam). On the other hand, it is possible to exploit the integrability of the nonlinear pump equation (2) to obtain a linear space of bound state solutions of (3), as was done in the bright ($\sigma=-1$) case [8]. Our job is to calculate these bound states and analyze their asymptotic behavior in the waveguide arms in order to prove the exact result that *all linear waveguides made from dark soliton collisions in slab geometry Kerr media function as perfect zero-crosstalk junctions at the frequency with which the waveguide was created.*

We will begin in Sec. II by explaining how the complete integrability of pump equation (2) can be used to produce rich (and perhaps complete) families of exact solutions of the associated linear problem (3). The essential idea is to set up a correspondence between the Lax eigenfunctions associated with ψ and solutions of (3). We then focus attention on the defocusing case of (2) and in Sec. III we present a construction of the N dark soliton solution of (2), obtaining as part of the same procedure the Lax eigenfunctions associated with dark soliton collisions. By the methods of Sec. II, these Lax eigenfunctions will generate solutions of the linear Schrödinger equation (3), including the bound states of the N dark soliton waveguide. Section IV is devoted to the asymptotic analysis of these bound states; this analysis proves that dark soliton waveguides are zero-crosstalk devices. Finally, in Sec. V we will discuss some possible applications of our results.

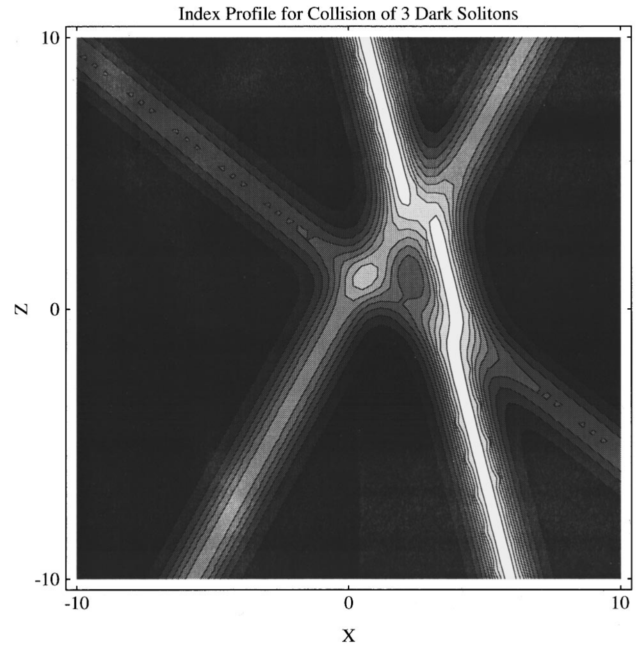

FIG. 1. The refractive index profile of a collision of three dark solitons. This profile may be considered as a six-port linear device, with three inputs at the bottom of the figure and three outputs at the top.

II. LAX EIGENFUNCTIONS AND SOLUTIONS OF THE LINEAR SCHRÖDINGER EQUATION

Our method of solving the linear problem (3), which is developed in greater detail in [9] and was practically applied in [8], is based upon the integrability of the nonlinear equation (2). This integrability simply means that (2) is the consistency condition for a pair of linear problems [ordinary differential equations with nonconstant coefficients depending on the field $\psi(X,Z)$] called a *Lax pair*. The simultaneous solution of the Lax pair is a two-component vector $\mathbf{u}=(u_1,u_2)^T$ which we will refer to as the *Lax eigenfunction*. The two linear problems making up the Lax pair for (2) are

$$\partial_X \mathbf{u} = \mathbf{L}\mathbf{u} = \begin{bmatrix} -i\lambda & \psi \\ \sigma\bar{\psi} & i\lambda \end{bmatrix} \mathbf{u}, \qquad (4)$$

$$i\partial_Z \mathbf{u} = \mathbf{B}\mathbf{u} = \begin{bmatrix} \lambda^2 + \frac{\sigma}{2}|\psi|^2 & i\lambda\psi - \frac{1}{2}\partial_X \psi \\ i\sigma\lambda\bar{\psi} + \frac{\sigma}{2}\partial_X \bar{\psi} & -\lambda^2 - \frac{\sigma}{2}|\psi|^2 \end{bmatrix} \mathbf{u}, \quad (5)$$

where λ is an arbitrary complex parameter. The compatibility condition for (4) and (5) is equivalent, regardless of the value of λ, to the cubic nonlinear Schrödinger equation (2). For each solution $\psi(X,Z)$ of (2), there is then a basis of two linearly independent Lax eigenfunctions parametrized by λ. Lax eigenfunctions and solutions ϕ of (3) are connected by

Proposition 1. Suppose that $\psi(X,Z)$ solves the nonlinear Schrödinger equation (2). Let $\mathbf{u}(X,Z;\lambda)$ be any corresponding Lax eigenfunction, for any complex λ. Then the function

$$\phi(X,Z) = u_1(X,Z;\lambda)\exp[-i(\lambda X + \lambda^2 Z)] \quad (6)$$

is a solution of the linear Schrödinger equation (3).

This proposition is proved and discussed in [9]. It establishes a clear correspondence between solutions of the linear problem (3) and solutions of the linear Lax pair (4) and (5). From one point of view, both of these linear problems (the linear Schrödinger equation and the Lax pair) are equally difficult, as they involve nonconstant coefficients through the given solution ψ of (2). However, the utility of the correspondence established in Proposition 1 is that many exact methods for finding functions $\psi(X,Z)$ that solve the nonlinear Schrödinger equation (2) also produce a basis of Lax eigenfunctions as a by-product. For classes of solutions ψ of (2) constructed by such methods, and, in particular, for the classes of bright and dark multisoliton solutions, the Proposition automatically provides a family, parametrized by an arbitrary complex number λ, of exact solutions to the linear Schrödinger equation (3).

III. DARK SOLITON COLLISIONS AND THEIR LAX EIGENFUNCTIONS

Now, and for the rest of the paper, we restrict attention to the defocusing case by setting $\sigma = +1$ in (2) and (3). Let us describe the construction of the solutions of (2) that correspond to the interaction of N dark solitons propagating on a given background field by a method that also gives the corresponding basis of simultaneous solutions to (4) and (5). The results of this section are not entirely new, with multiple dark soliton solutions being algebraic reductions of the Gelfand-Levitan-Marchenko inverse scattering equations first written down for this problem by Zakharov and Shabat [10], and with specific cases having been worked out, for example, by Blow and Doran [11]. The approach given here is similar to that described in [12]. It elucidates the essential role played by genus zero Riemann surfaces, as required by the nontrivial boundary conditions, and introduces the solitons in a completely algebraic manner. The Lax eigenfunction has a natural expression, which by the proposition will provide a convenient parametrization of solutions to the corresponding linear Schrödinger equation (3). We begin by specifying the background field that supports the dark solitons. Equation (2) has exact plane wave solutions of the form

$$\psi(X,Z) = a \exp[i(kX - \delta Z)], \quad (7)$$

where the complex amplitude a, wave number k, and shift in propagation constant δ are linked by the dispersion relation

$$\delta = \tfrac{1}{2}k^2 + |a|^2. \quad (8)$$

There is a corresponding simultaneous solution of the Lax pair (4) and (5) of the form

$$\mathbf{u}(X,Z,P) = \frac{a \exp\{\rho[X + (\lambda - k/2)Z]\}}{a + i(\lambda + k/2) + \rho} \times \begin{bmatrix} \exp[i(kX - \delta Z)/2] \\ [i(\lambda + k/2) + \rho]\exp[-i(kX - \delta Z)/2] \end{bmatrix}, \quad (9)$$

where ρ and λ are connected by the algebraic relation

$$\rho^2 + (\lambda + k/2)^2 = |a|^2. \quad (10)$$

The Lax eigenfunction $\mathbf{u}$ and the complex quantities ρ and λ are functions of a point P on the Riemann surface Γ defined by (10). It is convenient to refer to points P on Γ by ordered pairs $P = (\lambda, \rho)$, with the two coordinates related by (10). The functions $\lambda(P)$ and $\rho(P)$ then operate on the ordered pairs simply by projection. The Riemann surface Γ is a double covering of the complex λ plane. There are two points on Γ that correspond to $\lambda = \infty$ which we refer to as $\infty^{\pm}$ with the understanding that ∞^+ is the point near which $\rho = i\lambda + ik/2 + O(1/\lambda)$. With this notation, the plane wave solution (7) is recovered from its Lax eigenfunction $\mathbf{u}(X,Z,P)$ as the limit

$$\psi(X,Z) = 2i \lim_{P \to \infty^+} \lambda \exp[-i(\lambda X + \lambda^2 Z)] u_1(X,Z,P). \quad (11)$$

The formula (9) for $\mathbf{u}$ represents a basis of solutions of the Lax pair, since for all λ different from the two real branch points $\lambda = -k/2 \pm |a|$ (where the two sheets of Γ are identified) $\mathbf{u}$ amounts to two linearly independent vectors, one for each sheet. Away from the points $\infty^{\pm}$ on Γ, the components of $\mathbf{u}$ are meromorphic, both sharing a single pole that is fixed for all X and Z at a point P_0 on one of the sheets of Γ over the point $\lambda = \gamma_0 = -\mathrm{Im}(a) - k/2$. This point lies on the real λ axis, between the two branch points.

The solitons have yet to be introduced into the background field given by (7). To do this, we will transform the Lax eigenfunction (9) for the plane wave (7) in order to obtain the Lax eigenfunction for an interaction of N dark solitons propagating on the background field. The corresponding solution of (2) will be obtained from this transformed Lax eigenfunction by a formula similar to (11). The transformation method is closely related to the theory of Bäcklund transformations and singular (infinite period) limits of multiphase wave trains [12]. The idea is to introduce the degrees of freedom associated with the dark solitons by allowing the components of the vector function $\mathbf{u}$ to have more than just the one pole on Γ. Each pole in excess of the one already present in the plane wave Lax eigenfunction (9) will contribute one dark soliton to the field.

Let N be the desired number of solitons. Choose N points γ_i, $i = 1,\ldots,N$ in the *spectral interval* $I = [-k/2 - |a|, -k/2$

$+|a|]$ in which ρ is real valued. For each of the γ_i, select a particular point P_i on Γ so that $\lambda(P_i)=\gamma_i$. Now define the set of functions $g_i(P)$ on Γ, each of which has only a single pole at the point P_i. Up to scaling and addition of a constant, these functions are unique. An example is

$$g_i(P)=\{i[\lambda(P)-\lambda(P_i)]+[\rho(P)-\rho(P_i)]\}^{-1}, \quad (12)$$

although it will make no difference to the construction to select a different function of the form $\alpha g_i(P)+\beta$ for complex constants $\alpha(\neq 0)$ and β. Now, transform the Lax eigenfunction (9) by defining the new functions

$$\tilde{u}_1(X,Z,P)=\left\{1+\sum_{i=1}^{N} f_1^{(i)}(X,Z)[g_i(P) - g_i(\infty^-)]\right\}u_1(X,Z,P),$$

$$\tilde{u}_2(X,Z,P)=\left\{1+\sum_{i=1}^{N} f_2^{(i)}(X,Z)[g_i(P) - g_i(\infty^+)]\right\}u_2(X,Z,P). \quad (13)$$

These new functions will now have fixed poles at the points P_i, $i=1,\ldots,N$, as well as the pole at P_0 inherited from the functions u_1 and u_2. The locations of the corresponding zeros are controlled by the coefficient functions $f_1^{(i)}(X,Z)$ and $f_2^{(i)}(X,Z)$. These coefficients are determined by choosing N points λ_j in the spectral interval I and insisting that

$$\tilde{\mathbf{u}}(X,Z,P^+(\lambda_j))=\tilde{\mathbf{u}}(X,Z,P^-(\lambda_j)), \quad (14)$$

where $P^{\pm}(\lambda_j)$ are the two points on Γ that lie above $\lambda=\lambda_j$. This is a pair (one for each vector component) of systems of N linear algebraic equations for N unknowns, the solution of which results in expressions for $f_1^{(i)}(X,Z)$ and $f_2^{(i)}(X,Z)$ in terms of exponential functions making up $\mathbf{u}$.

Now, the transformed vector function $\tilde{\mathbf{u}}(X,Z,P)$ will also be a simultaneous solution of the Lax pair as long as the adjusted function

$$\tilde{\psi}(X,Z)=2i\lim_{P\to\infty^+}\lambda\,\exp[-i(\lambda X+\lambda^2 Z)]\tilde{u}_1(X,Z,P)$$
$$=\left\{1+\sum_{i=1}^{N} f_1^{(i)}(X,Z)[g_i(\infty^+)-g_i(\infty^-)]\right\}\psi(X,Z), \quad (15)$$

replaces ψ in the coefficients of the linear problems (4) and (5). By consistency, the function $\tilde{\psi}$ so constructed will thus solve (2). To review, the data involved in the construction of these solutions is the set $\{a,k,\lambda_1,\ldots,\lambda_N,P_1,\ldots,P_N\}$.

These solutions generally have singularities; however they represent (bounded) collisions of dark solitons if the data are chosen according to the following scheme. One first chooses an arbitrary plane wave background specified by the parameters a and k. The next choice is the set of numbers λ_j, and these can lie anywhere in the spectral interval I. The λ_j subdivide I into $N+1$ subintervals, one of which will contain

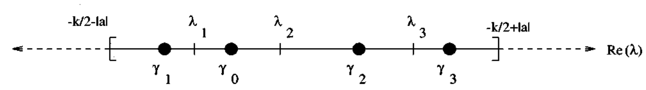

FIG. 2. The spectral interval I corresponding to a collision of three dark solitons.

the point γ_0 associated with the optical phase of the background plane wave. Given these choices, the restriction required to avoid singularities is that the poles P_i must be chosen so that each of the N other subintervals contains exactly one of the projections $\gamma_j=\lambda(P_i)$ (see Fig. 2). With these restrictions, the solution $\tilde{\psi}$ of (2) constructed from the data $\{a,k,\lambda_1,\ldots,\lambda_N,P_1,\ldots,P_N\}$ will correspond to the interaction of N dark solitons of the form

$$\psi_j(X,Z)=(\lambda_j+k/2+i\rho_j\,\tanh\{\rho_j[X+(\lambda_j-k/2)Z]\})$$
$$\times\exp[i(kX-\delta Z-\theta_0)], \quad (16)$$

where ρ_j denotes one of the values of ρ over $\lambda=\lambda_j$. The soliton slope is $k/2-\lambda_j$ and the contrast is $(\rho_j/|a|)^2$. The numbers λ_j thus determine the angles and contrasts of the individual solitons, while the pole locations P_i encode the centers of mass of the solitons at $Z=0$ and thus determine the geometry of the interaction region of the solitons. Note that for our purposes, we only need to construct the first component $\tilde{u}_1$ of the Lax eigenfunction for the dark soliton collision, so it will only be necessary to analyze a single N-by-N algebraic system of equations for the coefficients $f_1^{(i)}(X,Z)$.

IV. BOUND STATES AND ZERO-CROSSTALK WAVEGUIDES

We have now obtained the Lax eigenfunction corresponding to the interaction of N dark solitons by transforming that corresponding to the background field. The Proposition now establishes a connection between this Lax eigenfunction and solutions of the associated linear Schrödinger equation (3). In particular, the function

$$\phi(X,Z,P)=\tilde{u}_1(X,Z,P)\exp\{-i[\lambda(P)X+\lambda(P)^2 Z]\}, \quad (17)$$

solves (3) (in which we of course substitute for ψ the N soliton solution $\tilde{\psi}$). Whereas in the Lax pair, λ appears as an explicit parameter on which the Lax eigenfunction depends, the linear problem (3) has no such parameter. This means that (17) actually represents a large family of exact solutions of (3), with this family being parametrized by an arbitrary point P on the Riemann surface Γ. The family defined by (17) contains both bound and unbound solutions of (3), and of course by applying superposition to the solutions in this family one may obtain other solutions—perhaps even the general solution for initial ($Z=0$) conditions in an appropriate functional class.

In the theory of linear waveguides, a particularly important role will be played by the solutions $\phi_n(X,Z)=\phi(X,Z,(\lambda_n,\pm\rho_n))$ of the linear problem (3) which explicitly take the form

$$\phi_n(X,Z) = \frac{a\ \exp\{i[(k/2-\lambda_n)X-(\delta/2+\lambda_n^2)Z]\}}{a+i(\lambda_n+k/2)\pm\rho_n} \times\left\{1+\sum_{i=1}^{N} f_1^{(i)}(X,Z)\tilde{g}_i(\lambda_n,\pm\rho_n)\right\} \times\exp\{\pm\rho_n[X+(\lambda_n-k/2)Z]\}, \tag{18}$$

where we have introduced the abbreviated notation

$$\tilde{g}_i(P) \doteq g_i(P)-g_i(\infty^-). \tag{19}$$

Notice that as a result of the relations (14) and (17), this definition is independent of any consistent choice of the sign of $\pm\rho_n$. It turns out that the particular solutions ϕ_n are natural bound states of (3), and that these bound states describe the transmission of signals through the junction waveguide without any crosstalk.

To see this, analyze the asymptotic behavior of the solutions ϕ_n along straight lines in the (X,Z) plane. Choose a slope c and introduce the new variables $\chi=X-cZ$ and $\zeta=Z$. We will consider the limits $|\zeta|\to\infty$ for fixed χ. Now, studying the asymptotic behavior of ϕ_n requires first studying that of the coefficient functions $f_1^{(i)}(X,Z)$ defined by the N-by-N system of linear equations (14). In the variables ζ and χ, these equations take the concrete form

$$\frac{\{1+\Sigma_{i=1}^{N} f_1^{(i)}(\chi+c\zeta,\zeta)\tilde{g}_i(\lambda_j,-\rho_j)\}\exp(-\rho_j\chi)}{a+i\lambda_j+ik/2-\rho_j}\exp\{-\rho_j[c-(k/2-\lambda_j)]\zeta\} = \frac{\{1+\Sigma_{i=1}^{N} f_1^{(i)}(\chi+c\zeta,\zeta)\tilde{g}_i(\lambda_j,\rho_j)\}\exp(\rho_j\chi)}{a+i\lambda_j+ik/2+\rho_j}\exp\{\rho_j[c-(k/2-\lambda_j)]\zeta\}, \tag{20}$$

for $j=1,\ldots,N$. As long as $c\neq k/2-\lambda_j$ the asymptotic behavior for $|\zeta|$ large is easy to read off:

$$\lim_{|\zeta|\to\infty}\left\{1+\sum_{i=1}^{N} f_1^{(i)}(\chi+c\zeta,\zeta)\tilde{g}_i(\lambda_j,\pm\rho_j)\right\}\exp|\rho_j[c-(k/2-\lambda_j)]\zeta| = 0. \tag{21}$$

Now, along the line of slope c, the magnitude of the solution ϕ_n is

$$|\phi_n(\chi+c\zeta,\zeta)| = \frac{|a|\exp(\pm\rho_n\chi)}{|a+i(\lambda_n+k/2)\pm\rho_n|}\left|1+\sum_{i=1}^{N} f_1^{(i)}(\chi+c\zeta,\zeta)\tilde{g}_i(\lambda_n,\pm\rho_n)\right|\exp\{\pm\rho_n[c-(k/2-\lambda_n)]\zeta\}. \tag{22}$$

If $c\neq k/2-\lambda_n$, then it follows from (21) that for χ fixed $|\phi_n|$ vanishes as $|\zeta|\to\infty$. On the other hand, for $c=k/2-\lambda_n$ and χ fixed, $|\phi_n|$ approaches constant values $M_n^{\pm}(\chi)$ as $\zeta\to\pm\infty$.

The solution $\phi_n(X,Z)$ of (3) thus represents a beam that, when $Z\to-\infty$, is confined to the arm of the multisoliton waveguide corresponding to the dark soliton with slope $k/2-\lambda_n$ in the medium. In this limit, the beam profile is just the single mode of the dark soliton waveguide in isolation. The beam profile is altered somewhat as it enters the interaction region of the waveguide arms. However, when the beam emerges from the junction, in the limit $Z\to+\infty$, *it is again completely confined to the output arm having the same slope as the input arm*. The output beam profile is identical to the input profile, all the power having been conducted through the junction as if none of the other solitons had been present at all. *There is thus no crosstalk whatsoever among the linear channels joined by an index profile corresponding to a collision of N dark solitons in a planar Kerr medium*. This result holds for any number of dark solitons colliding at any distinct angles. As is the case with waveguides made from bright solitons [8], there is also no dependence on the precise geometry of the interaction encoded in the phase-shift parameters P_i; the remarkable asymptotic transmission properties of an N dark soliton waveguide are the same whether the solitons all collide more or less at the same place or collide only in pairs.

V. APPLICATIONS

The behavior of a waveguide made from a collision of dark solitons suggests applications to both high-bandwidth information processing and all-optical switching. In an information processing system, a large number N of independent channels could be physically brought together in an extremely small region without any mixing or loss among the channels, as long as the junction has an index profile made from a collision of dark solitons. This stands in contrast with the fact that, usually in integrated optics, the signal loses energy due to diffraction as it traverses a junction [13]. A zero-crosstalk junction made from dark solitons might find use in an optical computer, in the design of which practical constraints on physical dimensions play an important role. Regarding all-optical switching, it should be pointed out here that, in the special case of an index profile made from a two-soliton collision, the resulting waveguide can be characterized as a perfect half beat length device.

Of course, linear waveguides having complicated index profiles like those induced by dark soliton collisions cannot be fabricated by currently available techniques, although research continues on the possibility of "direct writing" of these devices with intense light in materials with memory [14]. Until these technologies are further developed, the results in this work may be used to improve the efficiency of step-index junctions, which may be easily fabricated by existing techniques. The idea is to find a step-index profile whose behavior closely approximates that of a multisoliton waveguide [15]. The hope is that simple heuristic rules may be discovered (for example, that waveguide arms should be flared or tapered somewhat as they join to form a junction) that may improve the performance of integrated optical devices.

The results of this paper may also be applied to problems in the time domain, where the coupled system (2) and (3) is a model for the copropagation of linear signals with a pump in an optical fiber with normal dispersion. In this context, the zero-crosstalk property suggests a new scheme for high-bandwidth communications in optical fibers, making use of so-called ''optical conveyor belts,'' which are discussed in detail elsewhere [16].

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