Exploiting discreteness for switching in waveguide arrays

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A new approach to multiport switching in arrays of nonlinear waveguides is proposed. Whereas other schemes have relied on suppressing the inherent transverse discreteness of these arrays, this approach takes advantage of that feature. One of the effects of discreteness is to keep intense beams trapped in a single waveguide for the length of the array. Switching may be achieved by use of a controlled perturbation to displace such a trapped beam in the transverse direction. This displacement is quantized to an integer number of waveguides, thus permitting unambiguous selection of the output channel.

All-optical signal processing with integrated nonlinear waveguide optics has many desirable features. In particular, it is possible to fabricate components that are small and capable of high-speed operation, limited in principle only by the turn-off time of the material nonlinearity.[1] One of the basic tasks of all-optical signal processing is switching, the ultimate goal being to achieve dynamic, fully controlled selection of one output channel among many. Here we consider the possibility of multiport switching in an array of N identical regularly spaced nonlinear waveguides, in which the stationary envelope of the electric field in the nth waveguide is governed by the discrete nonlinear Schrödinger equation

$$i\frac{\partial E_n}{\partial z} + (E_{n+1} - 2E_n + E_{n-1}) + |E_n|^2 E_n = 0, \qquad n = 1, 2, \ldots, N, \quad (1)$$

given here in dimensionless units.

The N-core coupler is a device described by Eq. (1),[2] in which switching is controlled by changing the power of a single-core input signal. Unfortunately, the power discrimination depends on the device length and decrease rapidly as the number of cores increases. In fact a complete power transfer into each of the output channels is no longer possible for more than three cores,[3] and if the number of cores exceeds five, the ability to control the switching is lost.[4] A new approach is thus required if more than five output channels are desired. One alternative is to use the collective properties of the array and suppress its inherent discreteness by operating it with low-intensity beams extending over several waveguides.[4] In this regime the array behaves essentially as a bulk medium and, correspondingly, beams are approximate spatial solitons of the continuous nonlinear Schrödinger equation. Thus a beam can propagate unhindered and emerge in a predictable region of the array, thereby selecting the output channel. In contrast to this approach, which we refer to as the continuum approach, we propose to exploit the discrete structure of the array by operating it at high intensities, at which the effects of discreteness become especially apparent.

The discrete nonlinear Schrödinger equation, Eq. (1), has two conserved quantities, the total power and the Hamiltonian. Thus it is generally only completely integrable for $N \leq 2$, which explains the predictability of the two-core coupler.[5] For more than two cores the model can exhibit chaotic behavior, which leads to, e.g., a high sensitivity of the three-core coupler to the device length.[6] However, even for large N, Eq. (1) allows beams to exist that can propagate through the array in a regular and predictable way. These beams have properties that differ in several ways from those of beams propagating in a bulk medium.

Let us briefly outline the beam properties that are important for multiport switching in arrays. For a recent review of discreteness effects in waveguide arrays, see Aceves *et al.*[7] Without the nonlinear term, Eq. (1) has linear plane-wave solutions of the form $\exp(ikn - i\beta z)$, where the propagation constant β is related to the wave number k by $\beta(k) = 4\sin^2(k/2)$. A packet of such plane waves, with wave numbers centered around k, will propagate in the array at an angle $\alpha(k)$, defined as $\tan(\alpha) \equiv \partial\beta/\partial k = 2\sin(k)$, and diffract subject to the linear diffraction coefficient $D(k) \equiv \frac{1}{2}\partial^2\beta/\partial k^2 = \cos(k)$. At low maximum intensity I_m the primary combined effect of nonlinearity and positive diffraction is to allow for steady and uniform propagation of beams at angle $\alpha(k)$. If terms of order $\mathcal{O}(I_m)$ are neglected, then the low-intensity solution to Eq. (1) can be found by use of multiple-scale techniques[8]:

$$E_n(z) \simeq I_m \operatorname{sech}\left\{\sqrt{I_m/2D}[n - \tan(\alpha)z]\right\} \times \exp\left\{i\left[kn - \left(\beta - \frac{1}{2}I_m\right)z + \theta\right]\right\}. \quad (2)$$

The diffraction coefficient $D(k)$ does not appear if the second difference in Eq. (1) is merely replaced by a second derivative and indicates correctly how this approximate solution breaks down for wave numbers above and close to the zero-diffraction wave number $|k| = \pi/2$. Thus $\alpha(\pi/2) \simeq 63.4°$ represents an upper limit of the propagation angle of beams in the array, a fact that is not considered in the continuum approach. The constant phase θ will play the role of a switching parameter.

If the intensity is permitted to increase, the propagation of beams at angles $\alpha \neq 0$ will be impeded by collisions with the periodic transverse structure of

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the array. In fact, at sufficiently high intensities any beam will quickly become completely trapped and forced to propagate at an angle $\alpha = 0$, regardless of its initial wave number. In this regime Eq. (1) has exact stationary solutions that are remarkably stable and very localized. These solutions are fundamentally different from the solitary beams of the form of relation (2) and play a key role in our approach to switching. No analytical formulas exist for such high-intensity solutions. However, in the limit of large intensity, asymptotic methods can be applied to yield the approximate solution $E_n(z) = \sqrt{I_m} U_n \exp(-i\beta z)$, where β and U_n are given by

$$\beta = 2 - I_m[1 + I_m{}^{-2} + \mathcal{O}(I_m{}^{-4})],$$
$$U_n = \begin{cases} 1 - \dfrac{1}{2} I_m{}^{-2} + \mathcal{O}(I_m{}^{-4}) & n = 0 \\ I_m{}^{-|n|}\left[1 + \dfrac{1}{2} I_m{}^{-2} + \mathcal{O}(I_m{}^{-4})\right] & n \neq 0 \end{cases}. \quad (3)$$

To improve this formula for finite values of I_m, we use Eqs. (3) as an initial guess in a Newton–Raphson iteration scheme. This yields numerically exact solutions, trapped beams, in the whole intensity range of interest.

It is thus apparent that two distinct intensity regimes exist in which beams can propagate through the array in a predictable way, and for switching applications it is important to identify them quantitatively. For arrays of length $L \leq 100$ the parameter regimes for angled beams are

$$I_m \leq 0.2, \quad |k| < \pi/2,$$

and for trapped beams they are

$$I_m \geq 1.7 \quad (4)$$

can be roughly estimated from numerical simulations of Eq. (1). Here and below, the number of waveguides N is chosen to be large enough to avoid boundary effects.

To exploit the effect of trapping for switching purposes the scheme must involve beams of maximum intensity greater than $I_m \approx 1.7$. As pointed out above, such a trapped beam cannot move transversely in the array by itself. However, it can be displaced by a sufficiently large perturbation. Let us consider two such perturbations:

(A). Imposing a linear phase gradient at the input. Here the initial beam profile is a numerically exact solution close to Eqs. (3) with maximum intensity $I_m^{\rm hi}$, which is then multiplied by $\exp(ikn)$ to initiate transverse propagation.

(B). Colliding with a low-intensity angled beam of the form of relation (2) with parameters ($I_m^{\rm lo}$, k, θ), launched Δn waveguides away from the trapped beam of maximum intensity $I_m^{\rm hi}$

Figure 1 shows representative examples of these two possibilities. In both cases the perturbation displaces the trapped beam in the transverse direction but only an integer number of waveguides, because of its high intensity and the subsequent strong trapping imposed by the discrete structure of the array. We propose to take advantage of this transversely quantized displacement in switching applications.

Let us look more closely at the switching process and how we can control it by producing many simulations of the type presented in Fig. 1. In each simulation the trapped beam is incident upon the array at $n = 0$ and has the approximate power $P_{\rm in} \equiv \sum_{-3}^{3} |E_n(0)|^2$. At the output $z = L$ we measure the position n_c as the waveguide of maximum intensity and define the contrast $C \equiv |E_{n_c}(L)|^2/(|E_{n_c-1}(L)|^2 + |E_{n_c}(L)|^2 + |E_{n_c+1}(L)|^2)$ and the power loss $\Delta P \equiv (P_{\rm in} - P_{\rm out})/P_{\rm in}$, where $P_{\rm out} \equiv \sum_{n_c-3}^{n_c+3} |E_n(L)|^2$ is the approximate output power.

Method A includes two control parameters, the maximum intensity $I_m^{\rm hi}$ and the wave number k. The length of the array and the number of waveguides in the array are fixed design parameters. Figure 2 shows the displacement of the trapped beam as function of $I_m^{\rm hi}$, with k as a parameter. Clearly, one can operate the array as a power-controlled switch by using the plateaus on the displacement curves. Thus switching among up to seven waveguides can be achieved for $k = -0.5$.

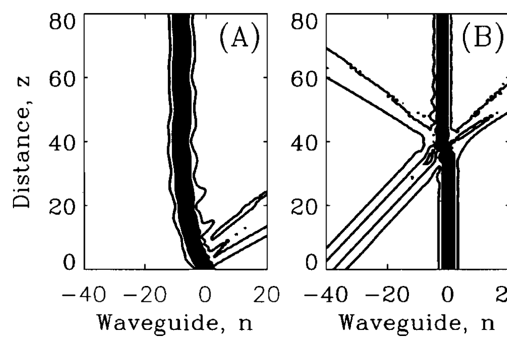

Fig. 1. Examples of methods (A) and (B) of displacing a trapped beam. Each plot shows the contour of the intensity $|E_n(z)|^2$ found by numerical integration of Eq. (1) with the initial condition as explained in the text. Common parameters: $L = 80$, $N = 101$, $I_m^{\rm hi} = 2.0$. Individual parameters: (A) $k = -0.52$, (B) $I_m^{\rm lo} = 0.2$, $k = 0.52$, $\theta = 0$, $\Delta n = 40$. The displacement is (A) eight waveguides and (B) two waveguides.

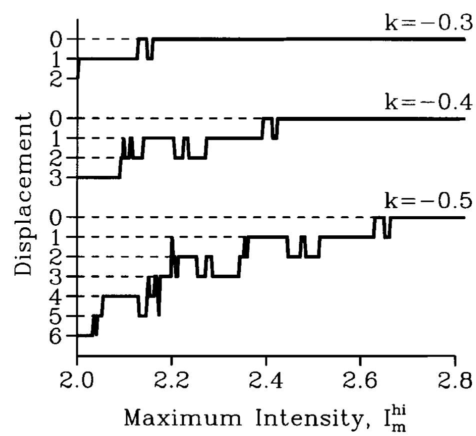

Fig. 2. Displacement of a trapped beam of maximum intensity $I_m^{\rm hi}$ after it has been given a linear phase gradient $\exp(ikn)$ at the input, as a function of $I_m^{\rm hi}$ for different values of k. The curves are the results of numerical simulation of Eq. (1) with the initial condition as explained in the text. Fixed parameters: $L = 40$, $N = 101$.

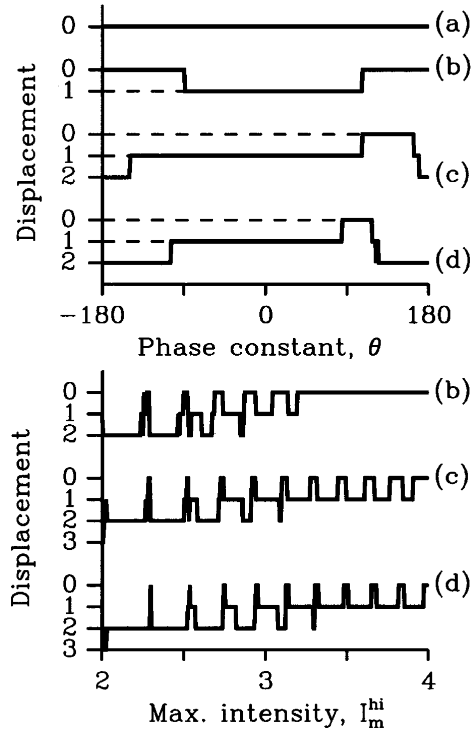

Fig. 3. Number of waveguides by which a trapped beam of maximum intensity I_m^{hi} is displaced after collision with an angled beam of maximum intensity I_m^{lo} of (a) 0.05, (b) 0.1, (c) 0.15, and (d) 0.2 as a function of (top) phase constant θ and (bottom) I_m^{hi}. The curves are the results of numerical simulation of Eq. (1) with the initial condition as explained in the text. Fixed parameters: $L = 80$, $N = 101$, $\Delta n = 40$, $k = 0.52$, and (top) $I_m^{\text{hi}} = 3$ and (bottom) $\theta = 0$.

Generally the displacement depends on the strength of the perturbation compared with the strength of the trapping exerted on the beam by the discrete structure of the array. Thus the higher the value of k and the lower the value of I_m^{hi}, the more the beam is displaced and the more sensitive the switching is on the control parameters. The contrast C of the output beam lies between 0.43 and 0.82, and the power loss is positive, varying between 3% and 11%. Note that in all simulations, including the one in Fig. 1(A), most of the loss in power is converted into a low-intensity beam that propagates through the array at an angle that increases with k. Method (A) for switching was also recently investigated by Aceves *et al.*[7]

Method (B) represents a switching process entirely different from Method (A). Because of the presence of the colliding beam, method (B) has four control parameters, the maximum intensities I_m^{hi} and I_m^{lo}, the wave number k, and the phase θ. The array in method (B) must be longer than in method (A) because the center of action of the perturbation is no longer at the input but is at some distance determined by k and the initial separation Δn [see Fig. 1(B)]. Thus we consider L, N, k, and Δn as fixed design parameters.

The top set of curves in Fig. 3 shows the displacement of the trapped beam as a function of θ for different values of I_m^{lo}. Because of the relatively high intensity of the trapped beam, $I_m^{\text{hi}} = 3$, switching can be achieved only among three waveguides, but the displacement curves are cleaner than in Fig. 2. Furthermore, the contrast is extremely good, ranging from 0.76 to 0.89. The power loss is now generally negative, $-10\% \le \Delta P \le 0\%$, and thus the input beam is amplified by the collision process. This amplification seems to be a general mechanism in discrete systems.[9] The bottom set of curves in Fig. 3 shows how power-controlled switching can be achieved with method (B). The oscillatory behavior of the curves is due to resonances between the intensity-dependent propagation constants of the two beams. The contrast C ranges from 0.59 to 0.91, and the input beam is still amplified by the collision, $-16\% \le \Delta P \le 0\%$.

Thus controlled switching can be achieved in arrays of nonlinear waveguides operated in the high-intensity regime, in which the properties of beams differ drastically from those of beams in bulk media. Correspondingly, the approaches to switching presented here are fundamentally different from the continuum approach.[4]

In the continuum approach the displacement of the input beam happens gradually over the whole length of the array. Thus the efficiency of the switching depends critically on the control of the angle, in that even slight variations change the waveguide in which the output signal is detected. The input contrast is $C = 0.45$ for the maximum intensity $I_m = 1.1$ used in Ref. 4. The output contrast will be somewhat less, in part because this value of I_m is outside the regime defined by Eq. (4), where beams resemble spatial solitons of a bulk medium.

By contrast, in both methods (A) and (B) the displacement of the trapped beam takes place in a small region in (n, z) space, which means that the efficiency of the switch can be insensitive to variations of the array length. The input beam is extremely localized, and this sharp contrast is maintained at the output, $0.43 \le C \le 0.91$. As noted in Ref. 10, maximizing the contrast is important for practical implementations. In addition, the two-beam method (B) provides the constant relative phase θ as a novel control parameter.

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