Homework Set 1

Solutions are due Friday, September 21st.

Problem 1. Let Y be the closed algebraic subset of $\mathbb{A}^3$ defined by the two polynomials $x^2 - yz$ and $xz - x$. Show that Y is a union of three irreducible components. Describe them and find their prime ideals.

Problem 2. Let X be a topological space.

i) Show that if Y is a subset of X (with the induced topology), then Y is irreducible if and only if its closure $\overline{Y}$ is irreducible.
ii) Suppose that X is Noetherian, and that Y is a subset X. Show that if $Y = Y_1 \cup \ldots \cup Y_r$ is the irreducible decomposition of Y, then $\overline{Y} = \overline{Y_1} \cup \ldots \cup \overline{Y_r}$ is the irreducible decomposition of $\overline{Y}$.

Problem 3. Note that we have a natural identification of $\mathbb{A}^2$ with the Cartezian product $\mathbb{A}^1 \times \mathbb{A}^1$. Show that the Zariski topology of $\mathbb{A}^2$ in *not* the product topology on $\mathbb{A}^1 \times \mathbb{A}^1$ of the two Zariski topologies on each $\mathbb{A}^1$.

Problem 4. Show that if f is a polynomial in $k[x_1, \ldots, x_n]$, then the corresponding hypersurface $V(f)$ is irreducible if and only if f has no distinct prime factors.