Problem session 2

Problem 1. For m and $n \geq 1$, let us identify $\mathbb{A}^{mn}$ with the set of all matrices $B \in M_{m,n}(k)$. Show that the set

$$M^r_{m,n}(k) := \{B \in M_{m,n}(k) \mid \mathrm{rk}(B) \leq r\}$$

is an algebraic subset of $M_{m,n}(k)$.

Problem 2. Consider the set Z consisting of those $(a, b, c, d) \in \mathbb{A}^4$ for which the rank of

$$\begin{pmatrix} a & b & c \\ b & c & d \end{pmatrix}$$

is ≤ 1. Show that Z is an irreducible closed algebraic subset of $\mathbb{A}^4$, and determine its ideal.

Problem 3. Show that a nonempty topological space X is irreducible if and only if for every two non-empty open subsets U and V, the intersection $U \cap V$ is nonempty. Deduce that an irreducible closed algebraic subset of $\mathbb{A}^n$ that is not a point, is not Haussdorff.

Problem 4. Is $\mathbb{C}^n$ with the usual Euclidean topology irreducible ?

Problem 5. Show that if X is a Noetherian topological space, and if Y is a subset of X (with the induced topology), then Y is Noetherian.