Problem session 5

Problem 1. Let X and Y be prevarieties, with Y affine.

i) Show that the canonical map
$$\mathrm{Hom}(X, Y) \to \mathrm{Hom}_{k-\mathrm{alg}}(\mathcal{O}(Y), \mathcal{O}(X))$$
given by $f \to f^\sharp$ is a bijection.

ii) Give an example to show that this may fail if Y is not affine.

Problem 2. Let X be a closed subset of $\mathbb{P}^n$. Show that X is irreducible if and only if the ideal $I(X) \subseteq k[x_0, \ldots, x_n]$ is prime.

Problem 3. Use the morphism $\mathbb{P}^1 \to \mathbb{P}^2$ given by $(x\colon y) \to (x^2\colon xy\colon y^2)$ to show that the homogeneous coordinate ring of a projective variety depends on the embedding in the projective space.

Problem 4. Let $f \in S = k[x_0, \ldots, x_n]$ be a homogeneous polynomial of positive degree.

1) Show that the open subset $D_+(f)$ of $\mathbb{P}^n$ is an affine variety with corresponding k-algebra $S_{(f)}$.
2) More generally, suppose that X is closed in $\mathbb{P}^n$. Show that $D_+(f) \cap X$ is an affine variety with corresponding ring $S(X)_{(f)}$.