Homework #1
Math 412, Winter 2014

1.1.9) Prove that if $a \in \mathbb{Z}$, then a^3 has exactly one of the forms $9k$, $9k+1$ or $9k+8$ for some integer k.

1.1.10) Suppose that n is a positive integer. Prove that two integers a and c have the same remainder when divided by n if and only if $a - c = nk$ for some integer k.

1.2.4b) Prove that if $a \mid b$, $a \mid c$ and $r, t \in \mathbb{Z}$, then $a \mid (br + ct)$.

1.2.20) Prove that if a and b are integers which are not both zero, then $(a, b) = (a, b + at)$ for all $t \in \mathbb{Z}$.

1.2.24) Suppose that $a, b, c \in \mathbb{Z}$ and that a and b are not both zero. Prove that the equation $ax + by = c$ has integer solutions if and only if $(a, b) \mid c$.

1.3.10) Suppose that p is an integer other than 0, 1 or -1. Prove that p is prime if and only if for every $a \in \mathbb{Z}$, either $(a, p) = 1$ or $p \mid a$.

1.3.16) Suppose that a and b are integers which are not both zero. Prove that $(a, b) = 1$ if and only if there is no prime integer p which divides both a and b.