Homework #2Math 412, Winter 2014

C.4) Let r be a real number such that $r \neq 1$. Prove that for every integer $n \geq 1$,

$$1 + r + \dots + r^{n-1} = \frac{r^n - 1}{r - 1}.$$

C.6) Prove that 3 is a divisor of $4^n - 1$ for any positive integer n.

2.1.12) Prove that if p is a prime integer and $p \ge 5$, then either [p] = [1] or p = [5] in \mathbb{Z}_6 .

2.1.20a) Prove or disprove: If $a^2 \equiv b^2 \pmod{n}$, then $a \equiv b \pmod{n}$ or $a \equiv -b \pmod{n}$.

b) Do part a) when n is prime.

2.2.14a) Solve the equation $x^2 + x = [0]$ in \mathbb{Z}_5 .

b) Solve the equation $x^2 + x = [0]$ in \mathbb{Z}_6 .

c) Prove that if p is prime, then the only solutions of $x^2 + x = [0]$ in \mathbb{Z}_p are [0] and [p-1].

2.3.10) Prove that every non-zero element of \mathbb{Z}_n is either a unit or a zero divisor, but not both.

2.3.12) Suppose that $a, b, n \in \mathbb{Z}$ and n > 1 and let d = (a, n). Prove that if [a]x = [b] has a solution in \mathbb{Z}_n , then $d \mid b$.