**Math 412, Winter 2014**

**Assignment #3: Due: Wednesday February 5**

Read sections 3.1, 3.2 and 3.3.

Complete, and turn in, problems 18, 20, and 42 in section 3.1, and problems 8, 18, 20, 22 (part a only), and 37 in section 3.2.

Complete, but do not turn in, problems 11, 17, 29, 33 and 34 in section 3.1, and problems 3, 13, 17, 21, 25 (see the hint in the back of the book), and 27 in section 32.

**First Midterm:** The first Midterm examination will be held in class on Wednesday February 12. It will cover the material in chapter 1, 2 and 3 and Appendix C.

**Homework problems to be turned in:**

3.1.18) Define a new multiplication on $\mathbb{Z}$ by setting $ab = 1$ for all $a, b \in \mathbb{Z}$. With this new multiplication and ordinary addition, is $\mathbb{Z}$ a ring? Prove your claim.

3.1.20) Show that $R = \{0, 3, 6, 9, 12, 15\}$ is a subring of $\mathbb{Z}_{18}$. Does $R$ have an identity? (Prove your claim)

3.1.42) A **division ring** is a (not-necessarily commutative) ring $R$ with identity $1_R \neq 0_R$ which satisfies axioms 11 and 12. Suppose that $R$ is a division ring and that $a$ and $b$ are non-zero elements of $R$.

a) Prove that if $bb = b$, then $b = 1_R$.

b) Prove that if $u$ is a solution of the equation $ax = 1_R$, then $u$ is also a solution of the equation $xa = 1_R$. (Remember that $R$ need not be commutative.)

3.2.8) Prove that if $R$ is a ring and $b$ is a fixed element of $R$, then $T = \{br \mid r \in R\}$ is a subring of $R$.

3.2.18) Let $a$ be a non-zero element of a ring $R$ with identity. Prove that if the equation $ax = 1_R$ has a solution $x = u$ and the equation $ya = 1_R$ has a solution $y = v$, then $u = v$.

3.2.20) Prove that if $R$ and $S$ are non-zero rings (meaning that each of them contains at least one non-zero element), then $R \times S$ contains zero divisors.

3.2.22a) Prove that if $ab$ is a zero divisor in a ring $R$, then $a$ or $b$ is a zero divisor in $R$.

3.2.37) Prove that if $R$ is a ring with identity $1_R$ and $a$ is not a zero divisor, then $ab = 1_R$ if and only if $ba = 1_R$.