Math 412, Winter 2014

Assignment #4: Due: Wednesday February 19

Read sections 4.1, 4.2 and 4.3

Complete, and turn in, problems 13, 15(part a only), 18 and 20 in section 4.1 and problems 8, 14 and 16 in section 4.2

Complete, but do not turn in, problems 1, 5, 6, and 9 in section 4.1, and problems 1, 3, 7, 9, and 15 in section 4.2.

Homework problems to be turned in:

4.1.13) Let R be a commutative ring. If $a_n \neq 0_R$ and

$$f(x) = a_0 + a_1 x + \cdots a_n x^n$$

is a zero divisor in $R[x]$, then a_n is a zero divisor in R.

4.1.15a) Let R be a commutative ring with identity and $a \in R$. If $a^3 = 0_R$, then $1_R + ax$ is a unit in $R[x]$. (Hint: Consider $1_R - ax + a^2x^2$.)

4.1.18) Suppose that R is a ring. Let $\phi : R[x] \to R$ be the map which takes each polynomial to its constant term in R, i.e.

$$\phi(a_0 + a_1 x + \cdots + a_n x^n) = a_0.$$

Show that ϕ is a surjective homomorphism of rings.

4.1.20) Let $D : \mathbb{R}[x] \to \mathbb{R}[x]$ be the derivative map

$$D(a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n) = a_1 + 2a_2 + \cdots n a_n x^{n-1}.$$

Is D a homomorphism of rings? Is it an isomorphism of rings?

4.2.8) Let F be a field and $f(x), g(x) \in F[x]$ not both zero and let $d(x)$ be the greatest common divisor of $f(x)$ and $g(x)$. Prove that if $h(x)$ is a common divisor of $f(x)$ and $g(x)$ of highest possible degree, then $h(x) = cd(x)$ for some non-zero $c \in F$.

4.1.14) Let F be a field and $f(x), g(x), h(x) \in F[x]$ with $f(x)$ and $g(x)$ relatively prime. Prove that if $f(x) \mid h(x)$ and $g(x) \mid h(x)$, then $f(x)g(x) \mid h(x)$.

4.1.16) Let F be a field and $f(x), g(x), h(x) \in F[x]$ with $f(x)$ and $g(x)$ relatively prime. Prove that the greatest common divisor of $f(x)h(x)$ and $g(x)$ is the same as the greatest common divisor of $h(x)$ and $g(x)$. (For simplicity, you may assume that $f(x)$, $g(x)$ and $h(x)$ are all non-zero.)