**Math 412 - Introduction to Abstract Algebra**

Homework 7

This homework assignment concerns sections 5.1-3 in the text. Please turn the following seven problems in on Wednesday, March 19.

1. (5.1.8) Prove of disprove. If $p(x)$ is relatively prime to $k(x)$ and $f(x)k(x) \equiv g(x)k(x) \mod p(x)$, then $f(x) \equiv g(x) \mod p(x)$.
2. (5.2.10) Let $F$ be a field and $p(x)$ a nonconstant polynomial in $F[x]$. Prove that $F^* = \{[a] : a \in F\}$ is a subring of $F[x]/p(x)$.
3. (5.2.11) Show that the ring $\mathbb{Q}[x]/(x^2)$ is not a field.
4. (5.2.16) Show that $\mathbb{Q}[x]/(x^2 - 2)$ is a field.
5. (5.3.2) Verify that $\mathbb{Q}(\sqrt{2}) = \{r + s\sqrt{2} : r, s \in \mathbb{Q}\}$ is a subfield of $\mathbb{R}$ and is isomorphic to $\mathbb{Q}[x]/(x^2 - 2)$.
6. (5.3.8) If $p(x)$ is an irreducible quadratic polynomial in $F[x]$, show that $F[x]/p(x)$ contains all roots of $p(x)$.
7. (5.3.10) Show that $\mathbb{Q}[x]/(x^2 - 2)$ and $\mathbb{Q}[x]/(x^2 - 3)$ are not isomorphic.

Please complete, but do not hand in exercises 5.1.1,4,13, 5.2.1,4,9,14, and 5.3.1,4,7,11.