Math 412 - Introduction to Abstract Algebra

Homework 8

This homework assignment concerns sections 7.1-3 in the text. Please turn the following seven problems in on Friday, April 4.

1. (7.1.28) Prove that each element of a finite group appears exactly once in each row and exactly once in each column of the operation table.

2. (7.2.21) Let G be a group and let $a \in G$. Then for all $m, m \in \mathbb{Z}$, we have $a^m a^n = a^{m+n}$ and $(a^m)^n = a^{mn}$.

3. (7.2.31,33) If $a, b \in G$ and $ab = ba$, prove that $(ab)^{|a||b|} = e$. Show this may be false if $ab \neq ba$. Moreover, if $a, b \in G$, $ab = ba$, and $|a|$ and $|b|$ are relatively prime, prove ab has order $|a||b|$.

4. (7.2.35) If $a, b \in G$, $b^6 = e$, and $ab = b^4 a$, prove that $b^3 = e$ and $ab = ba$.

5. (7.3.19) If G is an abelian group, prove the subset T of elements in G with finite order, is a subgroup of G. The subgroup T is called the **torsion** subgroup.

6. (7.3.33) Let G be a group and $a \in G$. The **centralizer** of a is the set $C(a) = \{g \in G : ga = ag\}$. Prove that $C(a)$ is a subgroup of G.

7. (7.3.27,39) Let H be a subgroup of G and, for $x \in G$, let $x^{-1}Hx$ denote the set $\{x^{-1}ax : a \in H\}$. Prove that $x^{-1}Hx$ is a subgroup of G. Prove the **normalizer** of H, defined as $N(H) = \{x \in G : x^{-1}Hx = H\}$ is a subgroup of G that contains H.

Please complete, but do not hand in exercises 7.1.4,10,16,30, 7.2.7,9,10,19, and 7.3.20,21.