Math 412 - Introduction to Abstract Algebra

Homework 9

This homework assignment concerns sections 7.3, 7.4, and 8.1 in the text. Please turn the following seven problems in on Wednesday, April 16.

- 1. (7.3.42) Let k be a positive divisor of the integer n. Prove that $H_k = \{a \in U_n : a \equiv 1 \mod n\}$ is a subgroup of U_n .
- 2. (7.3.53) Prove that $\mathbb{Z}_m \times \mathbb{Z}_n$ is cyclic iff m and n are relatively prime.
- 3. (7.4.19) Prove that a group G is abelian iff the map $f: G \to G$ given by $f(x) = x^{-1}$ is a homomorphism. In this case, prove it is an isomorphism.
- 4. (7.4.23) If G is an abelian group, prove the map $f : G \to G$ given by $f(x) = x^2$ is a homomorphism. Prove it is not a homomorphism for **every** nonabelian group.
- 5. (8.1.28) If n > 2, prove that the order of the group U_n is even.
- 6. (8.1.31) If G is a group of even order, prove that G contains an element of order two.
- 7. (8.1.32) If G is an abelian group of order 2n, with n odd, prove that G contains exactly one element of order two.

Please complete, but do not hand in exercises 7.3.38,44,45,55,56,7.4.20,32,47,54, and 8.1.29.