Math 525 - Probability

Homework 1

1. When we roll a single die, it is common to use the field $\{0,1\}^{\Omega}$ to define a probability measure. What is the smallest field that contains the outcome "roll a one" and the outcome "roll a two"?

2. Show that randomly selecting a rational number on the unit interval $(0,1)$ is a null event.

3. **Random Graphs:** Given n vertices $v_1, v_2, \ldots, v_n$ suppose we connect any two v_i, v_j $(i < j)$ with an edge with probability p. The resulting graph is called a random graph. When $n = 3$ and $p = 1/2$, show that the probability a random graph is connected is $1/2$. What is the probability a random graph with four vertices is connected for any p?

4. What is the probability that no two people in this class of 21 students share the same birthday? (Note: Assume there are 365 days in a year.)

5. Prove DeMorgan's laws: $(A \cup B)^c = A^c \cap B^c$ and $(A \cap B)^c = A^c \cup B^c$.

6. Use induction to prove the general inclusion exclusion property.

7. For events $E_1, E_2, \ldots, E_n$ prove that

$$P(E_1 \cap \cdots \cap E_n) = P(E_1)P(E_2|E_1)P(E_3|E_1 \cap E_2) \cdots P(E_n|E_1 \cap \cdots \cap E_{n-1}).$$