**Math 525 - Probability**

Homework 2

1. In poker, you are dealt a "flush" if all five cards that you receive have the same suit but are not in sequential order. You are dealt a "straight" if all five cards are in sequential order and at least two suits appear. Let us assume that the ace can be used as the highest or lowest card.

   What is the probability you are dealt a flush? What is the probability you are dealt a straight? What is the probability you are dealt a straight flush?

2. Use integration by parts to show that $n! = \int_0^\infty x^n e^{-x} dx$ for $n \geq 0$.

   Rewrite the integrand above as $e^{n \log x - x}$, substitute $x = ny$, and use an appropriate Taylor series for $\log x$ to prove Stirling's formula

   $$n! \sim n^n e^{-n} \sqrt{2\pi n}.$$

3. Prove Bayes' formula. This is problem 1.8.14 on page 22 of the text.

4. In the gambler's ruin problem prove that if we know $p_k = (p_{k+1} + p_{k-1})/2$, $p_0 = 1$, and $p_N = 0$ then the probability the gambler goes bankrupt is $1 - k/N$.

5. Complete problem 1.8.28 on page 24 of the text.

6. Suppose we flip a fair coin six times. Let the random variable $X$ count the number of heads that appear. Graph and describe the distribution function $F$ of $X$.

7. Suppose we construct a random graph on four vertices where two vertices are connected by an edge with probability $p$. Let the random variable $X$ count the number of connected components of the graph. Graph and describe the distribution function $F$ of $X$.

8. Complete exercises 2.1.2 and 2.1.4 on page 30 of the text.