Math 525 - Probability

Homework 3

1. Suppose we construct a random graph on four vertices with the probability any two vertices are connected by an edge being $p = 1/4$. Let the random variable X count the number of connected components and let the random variable Y count the size of the largest connected component.

   (a) Describe the distribution function F_X and F_Y corresponding to each random variable.

   (b) Describe the joint distribution function $F_{(X,Y)}$ of the random vector (X, Y).

   (c) Show that, in this case, we can recapture the marginal distribution functions F_X and F_Y from $F_{(X,Y)}$. That is, sum across the rows and columns of the joint distribution function above.

   (d) Calculate the median of each random variable and the random vector.

2. Prove this phenomenon in part (c) above is true for general F_X, F_Y, and $F_{(X,Y)}$.

3. Give an example to show the converse of part (d) above does not hold. That is, we cannot determine the joint distribution function $F_{(X,Y)}$ from F_X and F_Y alone.

4. Recall the uniform distribution on the interval $[\alpha, \beta]$ has a density function $f(x) = 1/(\beta - \alpha)$ for $\alpha \leq x \leq \beta$ and vanishes otherwise. The exponential distribution has a density function $f(x) = \lambda e^{-\lambda x}$ for $x \geq 0$ and vanishes otherwise.

   (a) Calculate the distribution function for each.

   (b) Calculate the median of each.

5. Use the MacLaurin series of e^x to calculate the distribution function corresponding to the standard normal density function $f(x) = (1/\sqrt{2\pi})e^{-x^2/2}$. Use this power series to show that the median is $x = 0$.