Math 256
Applied Honors Calculus IV: Differential Equations, Fall 2007

Homework Set 11
Due Monday, Nov 26, 2007

Reading/Study. Boyce and DiPrima 7.4, 7.5, 7.6, 7.7. The MATLAB command `pplane` for plotting phase diagrams (arrows) and phase portraits (with solution curves) of 2x2 linear systems. Figure out how to print so you can use them on HW. If you use MATLAB at home, the m-file for `pplane` is available on J. Polking's web page. A link is on our course web page. There is also a web based applet.

Be sure that you know how to plot direction vectors (a few) by hand.

Problems to Study. (Some will be assigned later.)

- Routine problems on systems. §7.5/1-17. §7.6/1-24.
- Phase portraits/diagrams for systems. §7.5/24-27. Use `pplane` too.
- Word problems. Boyce and DiPrima §7.1/17-23. §7.6/25-31. These are longer and more interesting.

Problems to Hand In.

- Systems from Boyce and DiPrima. §7.1/22. And add part **d.** Find the solution of the initial value from part **c**.
- Systems from Boyce and DiPrima. §7.5/2, 6, 29, 31(a bifurcation).
- Systems from Boyce and DiPrima. §7.6/1, 3.
- Saddles are not hyperbolas. Consider the saddle point system,

$$X' = \begin{bmatrix} 1 & 0 \\ 0 & -b \end{bmatrix} X, \qquad 0 < b.$$

  Equivalently,

$$x_1' = x_1, \qquad x_2' = -bx_2.$$

  Find a value of α so that the function

$$E(x_1, x_2) := x_1 (x_2)^\alpha$$

  is an integral of motion. That is, it is constant on solution curves. **Hint.** To do this without solving the differential equation compute the time derivative of E using the chain rule and use the differential equations for the time derivatives of the x_j. Choose α so the answer is zero.

  Conclude that for $b = 1$ the orbits are hyperbolas.

  Discussion. For $b \neq 1$ the orbits are solutions of an equation which is not quadratic so are **not** hyperbolas, though they look qualitatively like hyperbolas. You can reach this conclusion from the graphs of orbits. When $b \neq 1$ the orbits are **not** symmetric about the angle bisectors of the asymptotes. Hyperbolas **are** symmetric.