

Section 1.1, # 16. *Evaporation of a raindrop. Hint: evaporation refers to the change of volume over time, so you need to relate volume to surface area to solve this problem.*

Solution: Call the volume of the spherical raindrop V and the surface area S . We are told is that the evaporation rate dV/dt is proportional to S . So, there is some constant $k > 0$ so that

$$\frac{dV}{dt} = -kS$$

where we have the minus sign because evaporation amounts to a decrease of the total volume.

Next express S in terms of V . For a sphere of radius r , we have

$$V = \frac{4}{3}\pi r^3 \quad \text{and} \quad S = 4\pi r^2.$$

Use the first to express r as a function of V ,

$$r = \left(\frac{3V}{4\pi}\right)^{1/3}, \quad \text{so,} \quad S = 4\pi\left(\frac{3V}{4\pi}\right)^{2/3} = 6^{2/3}\pi^{1/3}V^{2/3}.$$

The differential equation becomes

$$\frac{dV}{dt} = -\tilde{k}V^{2/3}$$

where $\tilde{k} = 6^{2/3}\pi^{1/3}k$ is a new positive constant.

Here is a puzzle to make sure you've got this. If length is measured in centimeters and time in seconds, what are the units of k ? Compare with the "evaporation rate" in §2.5 problem 18.

In the third homework you will be asked to return to this problem to find out what it teaches you about the time it takes raindrops to evaporate, and ice droplets to melt (the same problem in disguise).