Math 256
Applied Honors Calculus IV: Differential Equations Fall 2007

Homework Set 3
Due Friday, September 28

Problems to Study

- Section 2.4, #14 and 16. *Determining the interval of existence of a solution, and how it depends on the initial condition.*
- Section 2.7, #1. *Trying out Euler's method. While in this case you can directly solve the differential equation, keep in mind that methods like Euler's method are most useful for differential equations where we cannot find an exact solution.*
- Solve the differential equation to find two distinct nonnegative solutions of the initial-value problem

$$y' = y^{1/3}\,, \qquad y(0) = 0$$

and check your solutions by differentiation. Why doesn't this result contradict the Existence-Uniqueness Theorem? Can you find any more solutions?

- Section 3.1, #5, 7, 11, 17, 22, and 25. *Second-order constant-coefficient differential equations. Look at problems 1–16 as needed.*

Problems to Hand In

- Raindrop revisited. Solve the equation for raindrop evaporation. Does the drop evaporate in finite time or does it take infinitely long? If finite, determine how long as a function of of the initial radius.
- Section 2.5, #20. *Harvesting of a renewable resource.*
- Section 2.5, #28. *Chemical reactions and limiting behavior.*
- Section 2.7, #20. *Understanding the convergence of Euler's method in a concrete example.*
- Drag forces revisited. Consider motion with drag forces, but without gravity acting.

(a) Suppose an object moves through a medium with a drag force proportional to its velocity v, so that $dv/dt = -kv$ with $k > 0$. Show that its velocity and position at time t are given by

$$v(t) = v_0 e^{-kt} \qquad \text{and} \qquad x(t) = x_0 + \frac{v_0}{k}\left(1 - e^{-kt}\right)\,.$$

Conclude that the object only manages to travel a finite distance before stopping, and find that distance.

(b) A more accurate model for drag forces when speeds are large is to assume that the force is proportional to the *square* of the velocity, so that $dv/dt = -kv^2$ for some $k > 0$. Show that

$$v(t) = \frac{v_0}{1 + v_0 kt} \qquad \text{and} \qquad x(t) = x_0 + \frac{1}{k}\ln(1 + v_0 kt)\,.$$

Find $\lim_{t\to\infty} x(t)$ in this case, and compare with part (a). In your comparison you should remark the very different behavior and attempt to *understand and explain* how it came about.

- Section 3.1, #8, 15, 21, and 23. *Second-order equations with constant coefficients.*