Math 256
Applied Honors Calculus IV: Differential Equations
Fall 2007

Homework Set 4
Due Monday, October 8, 2007

Recommended Reading: *Feynman Lectures on Physics* Volume I, Chapter 22 is an interesting account of the complex exponential. It derives Euler's formula using arithmetic and the law exponents and not infinite series. He does not *prove* that his algebraic sine and cosine functions are identical to those of geometry. His argument is very convincing.

Problems to Study:

- Determinants for 2× 2 systems of equations: Consider the equations

$$ax + by = f \qquad \text{and} \qquad cx + dy = g$$

  where a, b, c, and d, are given coefficients, f, and g are given numbers, and the unknowns are x and y. Solve these equations for x and y. Under what condition does your solution make sense for general choices of f and g? If this condition is satisfied, what are x and y if f and g are both zero? Suppose now that f and g are zero. Make a few concrete choices for the coefficients that violate the condition you found above. In each case, try to find a solution where x and y are not both zero.

- Section 3.2, #1–6. *Calculating the Wronskian.*
- Section 3.3, #1–8. *Determining linear independence. You may proceed either directly (checking whether the functions are proportional) or by using a Wronskian where appropriate.*
- Section 3.4, #1–6. *Elementary calculations using Euler's formula.*
- Section 3.5, #1–18. *Elementary calculations using annihilators or undetermined coefficients.*
- Euler's formula: Find a general procedure for finding a formula for $\sin(nx)$ for any n using Euler's formula? **Hint:** Do the first hand in problem first.

Problems to Hand In:

- Geometry Formulas from Algebra: Show all the steps of using Euler's formula to find an expression for $\cos(2x + y)$ in terms of $\sin(x)$, $\sin(y)$, $\cos(x)$, and $\cos(y)$. **Hint.** $\mathrm{Re}\, e^{i(x+x+y)}$.
- Section 3.2, #11. *Finding the interval of existence of solutions for second-order linear equations.*
- Section 3.3, #10, and 24. *Wronskians, Linear Independence, and Abel's Theorem.*
- Section 3.4, #11 and 12. *Complex roots of the characteristic equation.*
- Section 3.5, #8,16. *Repeated roots of the characteristic equation. Reduction of order.*
- Section 3.6, #31,32. *Repeated roots of the characteristic equation. Reduction of order.*