Two solutions for HW6

Identification of parameters.

For the underdamped spring
$$y'' + by' + cy = 0\,,$$

one has motions $e^{-at} \cos(\omega t - \delta)$.

The roots of the characteristic equation in this case are $a \pm i\omega$ with

$$a = b/2, \qquad \text{and}, \qquad \omega = \frac{\sqrt{4c - b^2}}{2}\,. \tag{1}$$

The frequency observation yields

$$\frac{\sqrt{4c - b^2}}{2} = 523. \tag{2}$$

The half life T is given by

$$e^{-aT} = 1/2, \qquad aT = \ln 2, \qquad a = \frac{\ln 2}{3}\,. \tag{3}$$

Inserting in (1) this yields
$$b = 2a = \frac{2\ln 2}{3}\,. \tag{4}$$

Plug (4) into (2) to find c.

Note that it is NOT true that $c = \omega^2$. This identity is nearly but not exactly satisfied.

Harvesting part b.

The differential equation is

$$\frac{dQ}{dt} = kQ(L - Q) - H\big(\sin(\pi t)\big)^2\,.$$

The harvesting term $-H\big(\sin(\pi t)\big)^2$ is nonpositive but vanishes when t is an even integer. Therefore
$$\max_t kQ(L - Q) - H\big(\sin(\pi t)\big)^2 = kQ(L - Q)\,.$$

In particular, for any $0 < Q < L$ the maximum is striclty positive.

On the other hand, for any $L \le Q < \infty$ the maximum is nonnegative.

Therefore the right hand side of the differential equation is nonpositive for $Q \ge L$ and on any larger half space it takes postive values.