Math 256
Applied Honors Calculus IV: Differential Equations
Fall 2007

Homework Set 8
Due Friday, November 2, 2007

Reading: Read sections 6.1, 6.2, 6.3, and 6.4.

Problems to Study:

- Section 6.1, #2 and 3. Continuity and piecewise continuity.
- Section 6.1, #21, 22, 23, and 24. Convergence of improper integrals.
- Laplace transforms. Perhaps with the help of the table on page 304, find the Laplace transforms $F(s)$ of the following functions.

  (i) $f(t) = t - 2e^{3t} + \sin(6t)$

  (ii) $f(t) = t^{3/2} - 1 + \cosh(5t) + \cos^2(t)$

  (iii) $f(t) = t\cos(at)$

  (iv) $f(t) = \sinh(bt) + e^{at}\cosh(bt)$

  (v) $f(t) = e^{at}\cos(bt)$ (Use the fact that Euler's formula implies $\cos(bt) = (e^{ibt} + e^{-ibt})/2$.)

  (vi) $f(t) = (t^4 - 7)e^{\pi t} + t^{3/2}e^{4t} + e^{-2t}\sin(3\pi t)$

  (vii) $f(t) = \begin{cases} 0\,, & 0 \le t < 1 \\ t^2 - 2t + 2\,, & t \ge 1 \end{cases}$ (Write in step-function notation first.)

- Inverse Laplace transforms. Find the inverse Laplace transforms $f(t)$ of the following functions.

  (i) $F(s) = \dfrac{2s-3}{s^2+7s+10}$

  (ii) $F(s) = \dfrac{2s-3}{s^2+2s+10}$

  (iii) $F(s) = \dfrac{1}{s^2+4s+4} + \dfrac{3}{2s-4}$

  (iv) $F(s) = \dfrac{1}{s^3-5s^2}$

  (v) $F(s) = \dfrac{(s-2)e^{-s}}{s^2-4s+3}$

Problems to Hand In:

- Measuring altitude using a pendulum clock. The differential equation for small-amplitude oscillations of a simple pendulum of length L is
$$L\,\theta''(t) + g\,\theta(t) = 0\,,$$
where $\theta(t)$ is the angle the pendulum makes with the vertical, and where the acceleration due to gravity at a distance R from the center of the planet is $g = GM/R^2$ with M being the mass of the planet and G being the universal gravitational constant.

(a) Two clocks with pendulums of lengths L_1 and L_2 are located at respective distances R_1 and R_2 from the center of the earth, and have respective periods of motion T_1 and T_2. Show that

$$\frac{T_1}{T_2} = \frac{R_1\sqrt{L_1}}{R_2\sqrt{L_2}}.$$

(b) Suppose a certain pendulum clock keeps perfect time in Paris, where the radius of the earth is $R = 3956$ miles. But when this same clock is taken to the equator of the earth on a ship, it is observed to run slower, losing 2 minutes and 40 seconds per day. Use this information to calculate the height above the center of the earth of sea level at the equator (this is called the equatorial bulge of the earth and is a centrifugal effect of the earth's rotation).

- Section 6.2, #12, 18, 20, and 22. Solving initial-value problems with the help of the Laplace transform.
- Section 6.4, #2, and 10. Solving initial-value problems with the help of the Laplace transform. Discontinuous forcing.