Solution of Chemical Reaction Problem, 94/28

The solutions uses the **Phase Line**. The key is the sign of the function

$$f(x) \;:=\; \alpha(p-x)(q-x)\,.$$

The concentrations p, q are unequal numbers belonging to the interval $[0, 1]$

The parameter α is positive.

Since $f(x)$ is a quadratic polynomial, the graph of $f(x)$ is the a parabola.

Since the coefficient of x^2 is equal to $\alpha > 0$, the parabola opens upward.

The parabola crosses the x-axis at the roots p, q. Thus $f(x) > 0$ for x below the two roots and for x above the two roots.

For x between the two roots, $f(x) < 0$.

Therefore, solutions to the left of the two roots increase. As $t \to \infty$ they converge to the lesser of the two roots. This solves part **a**, showing that the solution starting at 0, converges as $t \to \infty$, to the smaller of p and q.

The lesser of the two roots is stable.

Solutions to the right of the two roots increase. They diverge to infinity (in finite time).

For part **b**, the same analysis applies except that the interval between the roots degenerates to a single point. Still solutions starting to the left of the root, increase to the root as $t \to \infty$. Solutions starting to the right diverge to infinity (in finite time).

The explicit solution by formula is by separation of variables. The case of part **b** is easier as it does not require partial fractions for the integration.