Math 555, Fall 2001 Prof. J. Rauch

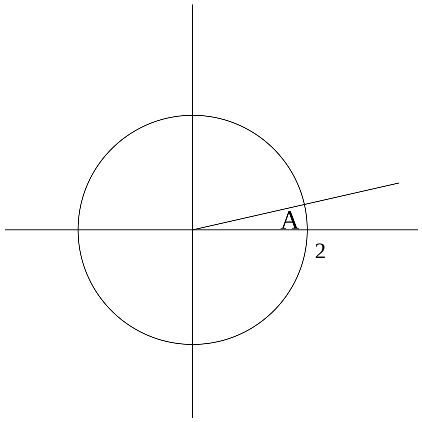
Midterm Exam October 30, 2001

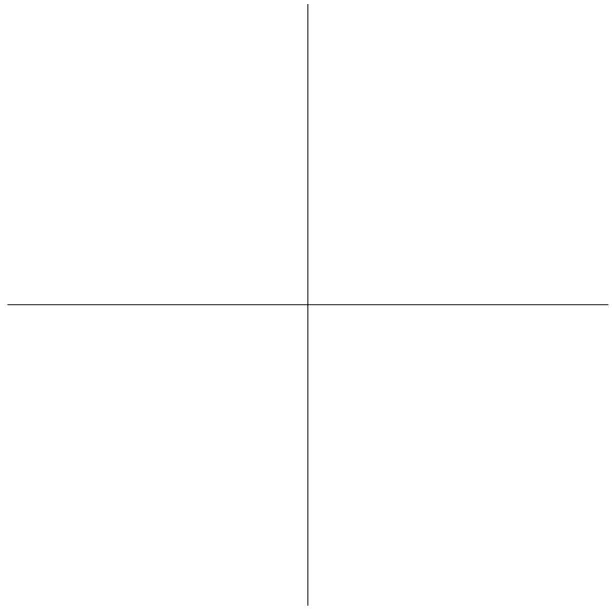
Instructions. 1. One side of an 8.5in. × 11in. sheet of notes from home. Closed book.
2. Show work and explain clearly.
3. There are six questions, one per page. 100 points total

1. (14 points) At what points $z = x + iy$ does the complex derivative, df/dz, exist when

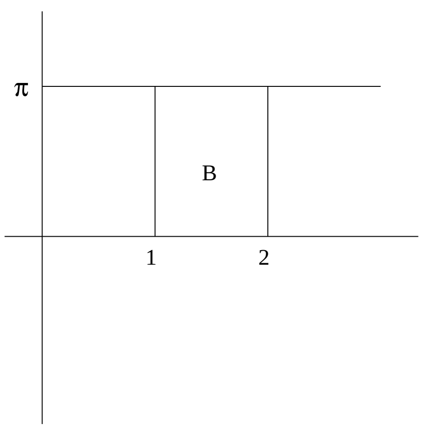
$$f(x+iy) \;=\; (x^2 - y^2) + i(x^2 + y^2)\,.$$

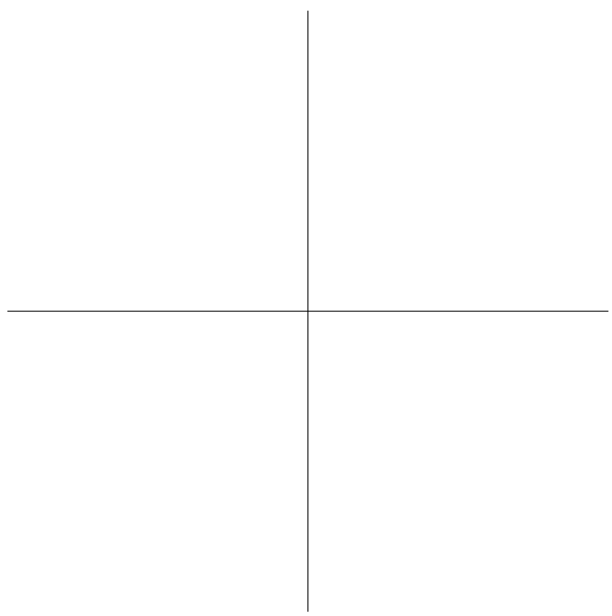
2. (18 points) **a.** Suppose that $f(z) = z^2$ and $A = \{z : |z| < 2, \quad 0 < \arg z < \pi/9\}$ is the angular sector of opening $\pi/9$ radians and radius 2 sketched on the left. Sketch the region $f(A) = \{f(z) : z \in A\}$ on the axes provided. Indicate important lengths and angles in the sketch.





b. Suppose that $g(z) = e^z$ and B is the rectangle $B = \{z : 1 \leq \text{Re}\, z \leq 2, \quad 0 \leq \text{Im}\, z \leq \pi\}$ sketched on the left. Sketch the region $g(B) = \{g(z) : z \in B\}$ on the axes provided. Indicate important lengths and angles in the sketch.





3. (14 points) Denote by C the arc of the parabola $y = x^2$ starting at $(0,0)$ and ending at $(1,1)$. Compute the line integral

$$\int_C x \; dz \, .$$

4. (14 points) Is there an analytic function $f(z)$ in the disc $\{|z| < 1\}$ with the property that

$$\frac{df}{dz} = |z|\,?$$

Explain why or why not.

5. (20 points) Compute using theorems of complex analysis each of the following integrals,

$$\int_{|z|=6} \frac{e^z}{(z-i\pi)^3}\, dz\,, \qquad \int_{|z|=2} \frac{e^z}{(z-i\pi)^3}\, dz\,.$$

The contours are oriented in the positive (counterclockwise) sense. State what results you are applying being sure that you mention the important hypotheses.

6. (20 points) Find the isolated singularities of the function

$$f(z) = \frac{e^{1/z}}{1-z}.$$

Compute the residue of f at each singularity.