**Math 555, Fall 2011** **J. Rauch**
**Homework 10** **Due November 22**

**1.** 189/21.

**2.** Use the method of Homework 9, Problems 1,2 to evaluate

$$\sum_1^\infty \frac{1}{1+n^2}\,.$$

**Hints.** Using a rectangle instead of a circle is a little easier.

**3.** Evaluate

$$\int_0^\infty \frac{z^{1/3}\log z}{z^2+1}\,dz\,,$$

where both $z^{1/3}$ and the logarithm are positive and real on $]0,\infty[$. **Ans.** $\pi^2/6$.

**4. i.** Find the image by the mapping $w = \sin z$ of the vertical half rays $\{x+iy \ : \ y \geq 0\}$ with $x = -\pi/2$, $x = 0$, and $x = \pi/2$.

**ii.** Show that $\sin z$ is a one to one function on the strip $-\pi/2 < x < \pi/2$, $0 \leq y \leq \infty$. **Hint.** Do **i** and **ii** with bare hands using $e^{iz}$.

**iii.** Use these two results to identify the image of the conformal map $w = \sin z$ of the strip from **ii**.

**5.** Find a conformal map from the sector $0 < \arg z < \pi/3$ to a disk of radius 1. The argument takes values in $]-\pi,\pi[$.

**6.** Find the image of the strip $0 < y < 1/(2c)$ under the transformation $w = 1/z$. Sketch the correspondence of the boundaries including orientations.

**7.** 122/25.