Math 555, Fall 2011 **J. Rauch**
Homework 11 **Due November 29**

Thanks to Thanksgiving, assignments with be accepted without penalty on Thurs. Dec. 1.

1. If F is a linear fractional transformation not equal to the identity show that there are at most two fixed points, that is values z so that $F(z) = z$.

2. Suppose that $-\infty < a < b < \infty$ and denote by $u(x, y)$ the unique bounded harmonic function in $y > 0$ that attains value 0 on $]-\infty, a[$, 1 on $]a, b[$, and, 0 on $]b, \infty[$. Find a harmonic conjugate of u in $y > 0$.

3. Find the unique steady state bounded temperature u in $y > 0$ that on the x-axis has value 0 on $]-\infty, -1[$, 1 on $]1, \infty[$, and, $]-1, 1[$ is insulated.

4. A steel plate is formed by cutting the unit disk into two pieces and discarding the lower piece. The cut is along the horizontal line through the point $z = e^{i\theta}$ with $0 < \theta < \pi$. The circular boundary of the resulting plate is kept at temperature $T = 1$. The horizontal boundary at $T = 0$. Find the unique bounded steady state temperature distribution.

5. 213/17. **Hint.** Show that for n large, $n!\alpha$ is an integer.

6. i. For integer $n \geq 1$ show that the irrotational, incompressible, planar fluid flow with complex potential $F(z) = z^n$ is tangent to the boundary of the wedge $0 < \arg z < 2\pi/n$ and each of its rotates by $k2\pi/n$ with $k \in \mathbb{Z}$.

ii. Sketch the streamlines. **Hint.** The streamlines satisfy $\operatorname{Im} z^n = c$. So z belongs to the image of a $\{\operatorname{Im} w = c\}$ by the appropriate branch of $z = w^{1/n}$.

iii. When n is even, show that the flow is tangent to the boundary of $\{y > 0\}$.

7. 136/10.