Math 555 Fall 2011 | Prof. J. Rauch
Homework 4 | Due October 6

**1.** 153/25.

**2.** 75/21.

**3.** 75/25.

**4.** 74/13.

**5.** If $f(z)$ is analytic on a neighborhood of $z = 0$ show that

$$\frac{f^{(n)}(0)}{n!}\, z^n \;=\; \sum_{j+k=n} \frac{\partial^j \partial^k f(0)}{\partial x^j \partial y^k}\, \frac{x^j\, y^k}{j!k!}\,.$$

**Discussion.** This shows that the Taylor series of $f$ at the origin in the sense of complex analysis is identical to the Taylor series as an infinitely differentiable function of $x, y$.

**6. i.** Use the known value of $\int_{-\infty}^{\infty} e^{-x^2/2}\, dx$ and a change of variables to derive a formula for $\int_{-\infty}^{\infty} e^{-ax^2/2}\, dx$ for $a > 0$.
**ii.** Derive formulas for

$$\int_{-\infty}^{\infty} x\, e^{-x^2/2}\, dx\,, \qquad \text{and,} \qquad \int_{-\infty}^{\infty} x^2\, e^{-x^2/2}\, dx\,,$$

by differentiating with respect to $a$. **Hint.** Leibniz' rule.

**7.** Let $S$ denote the square with vertices at $\pm 1$ and $\pm i$. For each of the following analytic functions, find the points of $S$ where $|f(z)|$ attains its maximum values.

$$\textbf{a}.\ f(z) \;=\; \sin z\,, \qquad \textbf{b}.\ f(z) \;=\; e^z\,, \qquad \textbf{c}.\ f(z) \;=\; \frac{1}{z-(1+i)}\,.$$

**Discussion.** These are all examples illustrating the maximum modulus principal.