Math 555 Fall 2011
Homework 5
Due October 13

1. 91/4.

2. 91/5. Give proof or couterexample.

3. 152/16.

4. 104/2.

5. 136/10a, b. Your explanation is important.

6. 137/21. Your explanation is important.

When computing Taylor series the formula $f^{(n)}(\underline{z})/n!$ for the coefficients is often not an efficient way to compute. It always pays to try to reduce calculatiions to known series like the exponential, trigonometric functions, and the geometric series. For example one should compute Taylor series of $e^z \sin z$ by multiplication. The series of e^z at $\underline{z}$ by writing $e^z = e^{\underline{z}} e^{z-\underline{z}}$. The next problem shows another good technique.

7. Compute the first terms of the Taylor series of $\tan z$ about $\underline{z} = 0$ as follows. Write the answer with undetermined coefficients a_j,

$$\tan z \;=\; \frac{\sin z}{\cos z} \;=\; a_0 + a_1 z + a_2 z^2 + \cdots .$$

Multiply through by $\cos z$. Expand sin and cos. Deterimine a_j for $j \le 5$ starting with a_0 so that the coefficients of the powers of z on the two sides coincide. **Discussion.** This method of undetermined coefficients is almost always faster than computing derivatives, *i.e.* $d^n \tan z/dz^n$.